Theoretical Prospects of Neutrinoless Double Beta Decay

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Abstract

The compelling experimental evidences for oscillations of solar and atmospheric neutrinos imply the existence of 3-neutrino mixing in vacuum. We briefly review the phenomenology of 3-ν mixing, and the current data on the 3-neutrino mixing parameters. The open questions and the main goals of future research in the field of neutrino mixing and oscillations are outlined. The predictions for the effective Majorana mass $|<m>|$ in $(\beta\beta)_{0\nu}-$decay in the case of 3-ν mixing and massive Majorana neutrinos are reviewed. The physics potential of the experiments, searching for $(\beta\beta)_{0\nu}-$decay and having sensitivity to $|<m>| \gtrsim 0.01$ eV, for providing information on the type of the neutrino mass spectrum, on the absolute scale of neutrino masses and on the Majorana CP-violation phases in the PMNS neutrino mixing matrix, is discussed.

1 Introduction

There has been a remarkable progress in the studies of neutrino oscillations in the last several years. The experiments with solar, atmospheric and reactor neutrinos [1, 2, 3, 4, 5, 6, 7] have provided compelling evidences for the existence of neutrino oscillations driven by nonzero neutrino masses and neutrino mixing. Evidences for oscillations of neutrinos were obtained also in the first long baseline accelerator neutrino experiment K2K [8].

The idea of neutrino oscillations was formulated in [9]. It was predicted in 1967 [10] that the existence of ν_e oscillations would cause a "disappearance" of solar ν_e on the way to the Earth. The hypothesis of solar ν_e oscillations, which (in one variety or another) were considered from ~1970 on as the most natural explanation of the observed [1] solar ν_e deficit (see, e.g., refs. [11, 12, 13, 14]), has received a convincing confirmation from the measurement of the solar neutrino flux through the neutral current reaction on deuterium by the SNO experiment [5], and by the first results of the KamLAND (KL) experiment [7]. The combined analysis of the solar neutrino data obtained by Homestake, SAGE, GALLEX/GNO, Super-Kamiokande (SK) and SNO experiments, and of the KL reactor $\bar{\nu}_e$ data [7], established the large mixing angle (LMA) MSW oscillations/transitions [12] as the dominant mechanism at the origin of the observed solar ν_e deficit (see, e.g., [15]). The Kamiokande experiment [16] provided the first evidences for oscillations of atmospheric ν_μ and $\bar{\nu}_\mu$, while the data of the SK experiment made the case of atmospheric neutrino oscillations convincing [6, 3]. Indications for ν-oscillations were reported by the LSND collaboration [17].

The latest contributions to these magnificent progress are the new SK data on the L/E-dependence of the μ-like atmospheric neutrino events [18], L and E being the distance traveled by neutrinos and the neutrino energy, and the new spectrum data of the KL and K2K experiments [19, 20]. For the first time the data exhibit directly the effects of the

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oscillatory dependence on L/E and E of the probabilities of ν-oscillations in vacuum [21]. As a result of these developments, the oscillations of solar ν_e, atmospheric ν_μ and $\bar{\nu}_\mu$, accelerator ν_μ (at $L \sim 250$ km) and reactor $\bar{\nu}_e$ (at $L \sim 180$ km), driven by nonzero ν-masses and ν-mixing, can be considered as practically established.

2   The Neutrino Mixing Parameters and $(\beta\beta)_{0\nu}$−Decay

The SK atmospheric neutrino and K2K data are best described in terms of dominant 2-neutrino $\nu_\mu \to \nu_\tau$ ($\bar{\nu}_\mu \to \bar{\nu}_\tau$) vacuum oscillations. The best fit values and the 99.73% C.L. allowed ranges of the atmospheric neutrino oscillation parameters read [3]:

$$|\Delta m_A^2| = 2.1 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_A = 1.0 \ ;$$
$$|\Delta m_A^2| = (1.3 - 4.2) \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_A \geq 0.85. \tag{1}$$

It should be noted that the signs of Δm_A^2 and of $\cos 2\theta_A$, if $\sin^2 2\theta_A \neq 1.0$, cannot be determined using the existing data.

Combined 2-neutrino oscillation analyses of the solar neutrino and the new KL spectrum data show [19, 22] that the $\nu_\odot$-oscillation parameters lie in the low-LMA region : $\Delta m_\odot^2 = (7.9^{+0.6}_{-0.5}) \times 10^{-5}$ eV^2, $\tan^2 \theta_\odot = (0.40^{+0.09}_{-0.07})$. The high-LMA solution (see, e.g., [15]) is excluded at $\sim 3.3\sigma$. Maximal $\nu_\odot$-mixing is ruled out at $\sim 6\sigma$; at 95% C.L. one finds $\cos 2\theta_\odot \geq 0.28$ [22], which has important implications (see further). One also has: $\Delta m_\odot^2 / |\Delta m_A^2| \sim 0.04 \ll 1$.

The evidences for ν-oscillations obtained in the solar and atmospheric neutrino and KL and K2K experiments imply the existence of 3-ν mixing in the weak charged lepton current:

$$\nu_{lL} = \sum_{j=1}^{3} U_{lj} \, \nu_{jL}, \quad l = e, \mu, \tau, \tag{2}$$

where ν_{lL} are the flavour neutrino fields, ν_{jL} is the field of neutrino ν_j having a mass m_j and U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [9, 23], $U \equiv U_{\rm PMNS}$. All existing ν-oscillation data, except the data of LSND experiment [17], can be described assuming 3-ν mixing in vacuum and we will consider this possibility in what follows [1].

The PMNS matrix can be parametrized by 3 angles, and, depending on whether the massive neutrinos ν_j are Dirac or Majorana particles, by 1 or 3 CP-violation (CPV) phases [26, 27]. In the standardly used parameterization (see, e.g., [28]), $U_{\rm PMNS}$ has the form:

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \text{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}) ,$$
$$\tag{3}$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$, the angles $\theta_{ij} = [0, \pi/2]$, $\delta = [0, 2\pi]$ is the Dirac CPV phase and α_{21}, α_{31} are two Majorana CPV phases [26, 27]. One can identify $\Delta m_\odot^2 = \Delta m_{21}^2 > 0$. In this case $|\Delta m_A^2| = |\Delta m_{31}^2| \cong |\Delta m_{32}^2| \gg \Delta m_{21}^2$, $\theta_{12} = \theta_\odot$, $\theta_{23} = \theta_A$. The angle θ_{13} is limited by the data from the CHOOZ and Palo Verde experiments [29]. The presently existing

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[1]In the LSND experiment indications for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations with $(\Delta m^2)_{\rm LSND} \simeq 1$ eV^2 were obtained. The minimal 4-ν mixing scheme which could incorporate the LSND indications for $\bar{\nu}_\mu$ oscillations is strongly disfavored by the data [24]. The ν-oscillation explanation of the LSND results is possible assuming 5-ν mixing [25]. The LSND results are being tested in the MiniBooNE experiment [25].

atmospheric neutrino data is essentially insensitive to θ_{13} satisfying the CHOOZ limit [3]. The probabilities of survival of solar ν_e and reactor $\bar{\nu}_e$, relevant for the interpretation of the solar neutrino, KL and CHOOZ data, depend in the case of interest, $|\Delta m^2_{31}| \gg \Delta m^2_{21}$, on θ_{13}:

$$P^{3\nu}_{\text{KL}} \cong \sin^4 \theta_{13} + \cos^4 \theta_{13} \left[1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{21} L}{4E} \right], \quad P^{3\nu}_{\text{CHOOZ}} \cong 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31} L}{4E},$$

$$P^{3\nu}_{\odot} \cong \sin^4 \theta_{13} + \cos^4 \theta_{13} \; P^{2\nu}_{\odot}(\Delta m^2_{21}, \theta_{12}; N_e \cos^2 \theta_{13}),$$

where $P^{2\nu}_{\odot}$ is the solar ν_e survival probability [30] corresponding to 2-ν oscillations driven by Δm^2_{21} and θ_{12}, in which the solar e^- number density N_e is replaced by $N_e \cos^2 \theta_{13}$ [31], $P^{2\nu}_{\odot} = \bar{P}^{2\nu}_{\odot} + P^{2\nu}_{\odot \, \text{osc}}$, $P^{2\nu}_{\odot \, \text{osc}}$ being an oscillating term [30] and

$$\bar{P}^{2\nu}_{\odot} = \tfrac{1}{2} + (\tfrac{1}{2} - P') \cos 2\theta^m_{12}(t_0) \cos 2\theta_{12},$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2_{21}}{2E} \sin^2 \theta_{12}} - e^{-2\pi r_0 \frac{\Delta m^2_{21}}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2_{21}}{2E}}}. \tag{4}$$

Here $\bar{P}^{2\nu}_{\odot}$ is the average probability [32, 30], P' is the "double exponential" jump probability [30] and r_0 is the "running" scale-height of the change of N_e along the ν-trajectory in the Sun ^{2} [30, 33, 34]. In the LMA solution region $P^{2\nu}_{\odot \, \text{osc}} \cong 0$ [34]. Using the expressions for $P^{3\nu}_{\text{KL}}$, $P^{3\nu}_{\text{CHOOZ}}$ and $P^{3\nu}_{\odot}$ given above, the 3σ allowed range of $|\Delta m^2_{\text{A}}|$ from [3], and performing a combined analysis of the solar neutrino, CHOOZ and KL data, one finds [22]:

$$\sin^2 \theta_{13} < 0.055, \quad 99.73\% \text{ C.L.} \tag{5}$$

Similar constraint is obtained from a global 3-ν oscillation analysis of solar, atmospheric and reactor neutrino data [24, 36]. A combined 3-ν oscillation analysis of the solar neutrino, CHOOZ and KL data shows also [22] that for $\sin^2 \theta_{13} \lesssim 0.02$ the allowed ranges of Δm^2_{21} and $\sin^2 \theta_{21}$ do not differ substantially from those derived in the 2-ν oscillation analyzes (see, e.g., ref. [19]). The best fit values, e.g., read ^{3} [22]:

$$\Delta m^2_{21} = 8.0 \times 10^{-5} \text{ eV}^2, \quad \sin^2 \theta_{21} = 0.28. \tag{6}$$

In Fig. 1 we show the allowed regions in the Δm^2_{21} -$\sin^2 \theta_{12}$ plane, obtained in a 3-ν oscillation analysis of the solar neutrino, KL and CHOOZ data for few fixed values of $\sin^2 \theta_{13}$.

As we have seen, the fundamental parameters characterizing the 3-neutrino mixing are: i) the 3 angles θ_{12}, θ_{23}, θ_{13}, ii) depending on the nature of ν_j - 1 Dirac (δ), or 1 Dirac + 2 Majorana ($\delta, \alpha_{21}, \alpha_{31}$), CPV phases, and iii) the 3 neutrino masses, m_1, m_2, m_3. It is convenient to express the two larger masses in terms of the third mass and the measured $\Delta m^2_{\odot} = \Delta m^2_{21} > 0$ and Δm^2_{A}. We have remarked earlier that the atmospheric neutrino and K2K data do not allow one to determine the sign of Δm^2_{A}. This implies that if we identify Δm^2_{A} with $\Delta m^2_{31(2)}$ in the case of 3-neutrino mixing, one can have $\Delta m^2_{31(2)} > 0$ or

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^{2}The analyses and the extensive numerical studies performed in [33, 34] show that expression (4) for $\bar{P}^{2\nu}_{\odot}$ provides a high precision description of the average solar ν_e survival probability in the Sun for any values of Δm^2_{21} and θ_{12} (the relevant error does not exceed $\sim$(2-3)%), including the values from the LMA region. The results obtained recently in [35] imply actually that the use of the double exponential expression for P' for description of the LMA transitions brings an imprecision in $\bar{P}^{2\nu}_{\odot}$ which does not exceed $\sim 10^{-6}$.

^{3}The best fit value of $\sin^2 \theta_{13} = 0.004$ is different from zero but not at statistically significant level [22].
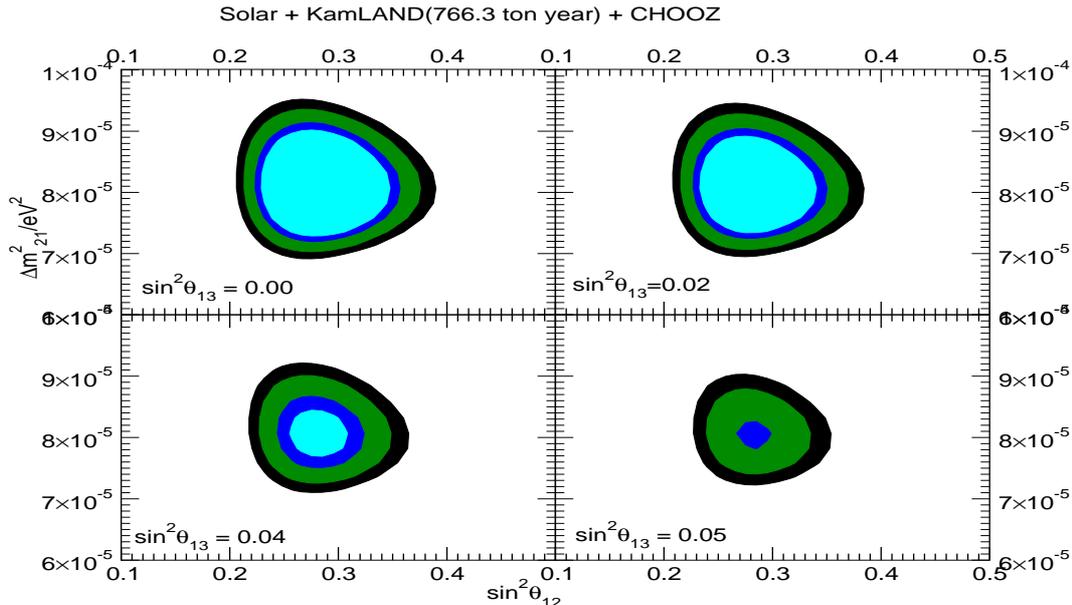
Figure 1: The 90%, 95%, 99% and 99.73% C.L. allowed regions in the Δm_{21}^2 -$\sin^2\theta_{12}$ plane, obtained in a 3-ν oscillation analysis of the solar neutrino, KL and CHOOZ data [22].

$\Delta m_{31(2)}^2 < 0$. The two possible signs of Δm_A^2 correspond to two types of ν-mass spectrum:

– *with normal hierarchy,* $m_1 < m_2 < m_3$, $\Delta m_A^2 = \Delta m_{31}^2 > 0$, $m_{2(3)} = (m_1^2 + \Delta m_{21(31)}^2)^{\frac{1}{2}}$, and

– *with inverted hierarchy* [4], $m_3 < m_1 < m_2$, $\Delta m_A^2 = \Delta m_{32}^2 < 0$, $m_2 = (m_3^2 - \Delta m_{32}^2)^{\frac{1}{2}}$, etc.

The neutrino mass spectrum can also be

– *Normal Hierarchical (NH):* $m_1 \ll m_2 \ll m_3$, $m_2 \cong (\Delta m_\odot^2)^{\frac{1}{2}} \sim 0.009$ eV, $m_3 \cong |\Delta m_A^2|^{\frac{1}{2}}$; or

– *Inverted Hierarchical (IH):* $m_3 \ll m_1 < m_2$, with $m_{1,2} \cong |\Delta m_A^2|^{\frac{1}{2}} \sim 0.045$ eV; or

– *Quasi-Degenerate (QD):* $m_1 \cong m_2 \cong m_3 \cong m_0$, $m_j^2 \gg |\Delta m_A^2|$, $m_0 \gtrsim 0.20$ eV.

Neutrino oscillation experiments allow to determine differences of squares of neutrino masses, but not the absolute values of the masses, or $min(m_j)$. Information on the absolute scale of ν- masses can be derived in ^{3}H β-decay experiments [39, 40, 41] and from cosmological and astrophysical data (see, e.g., ref. [42]). The currently existing most stringent upper bounds on the $\bar{\nu}_e$ mass were obtained in the Troitzk [40] and Mainz [41] experiments:

$$m_{\bar{\nu}_e} < 2.3 \text{ eV} \quad (95\% \ C.L.). \tag{7}$$

We have $m_{\bar{\nu}_e} \cong m_{1,2,3}$ in the case of QD ν-mass spectrum. The KATRIN experiment [41] is planned to reach a sensitivity to $m_{\bar{\nu}_e} \sim 0.20$ eV, i.e., to probe the region of the QD spectrum. The CMB data of the WMAP experiment were used to obtain the upper limit [43]:

$$\sum_j m_j < (0.7 - 2.0) \text{ eV} \quad (95\% \ C.L.), \tag{8}$$

where we have included a conservative estimate of the uncertainty in the upper limit (see, e.g., [42]). The WMAP and future PLANCK experiments can be sensitive to $\sum_j m_j \cong 0.4$

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[4]In the convention we use (called A), the neutrino masses are not ordered in magnitude according to their index number: $\Delta m_{31}^2 < 0$ corresponds to $m_3 < m_1 < m_2$. We can also always number the neutrinos with definite mass in such a way that [37] $m_1 < m_2 < m_3$. In this convention (called B), we have in the case of inverted hierarchy spectrum: $\Delta m_\odot^2 = \Delta m_{32}^2$, $\Delta m_A^2 = \Delta m_{31}^2$. Convention B is used, e.g., in [28, 38].

eV. Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow one to determine $(m_1 + m_2 + m_3)$ with an uncertainty of $\delta \sim (0.04 - 0.10)$ eV (see [42] and the references quoted therein).

The type of neutrino mass spectrum, i.e., $sgn(\Delta m^2_{31})$, can be determined by studying oscillations of neutrinos and antineutrinos, say, $\nu_\mu \leftrightarrow \nu_e$ and $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$, in which matter effects are sufficiently large. This can be done in long base-line ν-oscillation experiments (see, e.g., [44]). If $\sin^2 2\theta_{13} \gtrsim 0.05$ and $\sin^2 \theta_{23} \gtrsim 0.50$, information on $sgn(\Delta m^2_{31})$ might be obtained in atmospheric neutrino experiments by investigating the effects of the subdominant transitions $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ of atmospheric neutrinos which traverse the Earth [45]. For $\nu_{\mu(e)}$ (or $\bar{\nu}_{\mu(e)}$) crossing the Earth core, new type of resonance-like enhancement of the indicated transitions takes place due to the *(Earth) mantle-core constructive interference effect (neutrino oscillation length resonance (NOLR))* [46] [5]. As a consequence of this effect [6] the corresponding $\nu_{\mu(e)}$ (or $\bar{\nu}_{\mu(e)}$) transition probabilities can be maximal [47]. For $\Delta m^2_{31} > 0$, the neutrino transitions $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ are enhanced, while for $\Delta m^2_{31} < 0$ the enhancement of antineutrino transitions $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ takes place, which might allow to determine $sgn(\Delta m^2_{31})$.

After the spectacular experimental progress made in the studies of ν-oscillations, further understanding of the structure of neutrino mixing, of its origins and of the status of CP-symmetry in the lepton sector, requires a large and challenging program of research to be pursued in neutrino physics. The main goals of this research program should include [50]:
– High precision measurement of neutrino mixing parameters which control the solar and the dominant atmospheric neutrino oscillations, Δm^2_{21}, θ_{21}, and Δm^2_{31}, θ_{23}.
– Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, θ_{13} - the only small mixing angle in U_{PMNS}.
– Determination of the $sgn(\Delta m^2_{31})$ and of the type of ν-mass spectrum $(NH, IH, QD, \text{etc.})$.
– Determination or obtaining significant constraints on the absolute scale of ν-masses.
– Determination of the nature–Dirac or Majorana, of massive neutrinos ν_j.
– Establishing whether the CP-symmetry is violated in the lepton sector a) due to the Dirac phase δ, and/or b) due to the Majorana phases α_{21} and α_{31} if ν_j are Majorana particles.
– Searching with increased sensitivity for possible manifestations, other than flavour neutrino oscillations, of the non-conservation of the individual lepton charges L_l, $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. decays.
– Understanding at fundamental level the mechanism giving rise to neutrino masses and mixing and to L_l-non-conservation. This includes understanding the origin of the patterns of ν-mixing and ν-masses suggested by the data. Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of new symmetry of particle interactions? Is there any relations between quark mixing and neutrino mixing? Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$? Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS}? Progress in the theory of neutrino mixing might also lead, in particular, to a better understanding of the origin of baryon asymmetry of the Universe [57].

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[5]For the precise conditions of the mantle-core (NOLR) enhancement see [46, 47].

[6]The Earth mantle-core (NOLR) enhancement of neutrino transitions differs [46] from the MSW one. It also differs [46, 47] from the parametric resonance mechanisms of enhancement discussed in [48]: the conditions of enhancement found in [48] are not realized for the neutrino transitions in the Earth. In [49] it is erroneously concluded that the $\nu_{\mu(e)} \leftrightarrow \nu_{e(\mu)}$ transitions of atmospheric neutrinos crossing the Earth core cannot be enhanced by the interplay of the transitions in the Earth mantle and of those in the Earth core.

The mixing angles, θ_{21}, θ_{23} and θ_{13}, the Dirac CPV phase δ and Δm^2_{21} and Δm^2_{31} can, in principle, be measured with a sufficiently high precision in ν-oscillation experiments (see, e.g., [44, 51]). These experiments, however, cannot provide information on the ν-mass scale and on the nature of massive neutrinos ν_j, they are insensitive to the Majorana CPV phases $\alpha_{21,31}$ [26, 52]. Establishing whether ν_j have distinct antiparticles (Dirac fermions) or not (Majorana fermions) is of fundamental importance for understanding the underlying symmetries of particle interactions and the origin of ν-masses. If ν_j are Majorana fermions, getting experimental information about the Majorana CPV phases in U_{PMNS} would be a remarkably challenging problem [37, 28, 53, 54, 55]. The phases $\alpha_{21,31}$ can affect significantly the predictions for the rates of (LFV) decays $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. in a large class of supersymmetric theories with see-saw mechanism of ν-mass generation (see, e.g., [56]). Majorana CPV phases might be at the origin of baryon asymmetry of the Universe [57].

In the present article we will review the potential contribution the studies of neutrinoless double beta $((\beta\beta)_{0\nu}-)$ decay of even-even nuclei, $(A, Z) \to (A, Z + 2) + e^- + e^-$, can make to the program of research outlined above. The $(\beta\beta)_{0\nu}$-decay is allowed if the neutrinos with definite mass are Majorana particles (for reviews see, e.g., [13, 58, 59, 60]). Let us recall that the nature - Dirac or Majorana, of the massive neutrinos ν_j, is related to the symmetries of particle interactions. Neutrinos ν_j will be Dirac fermions if the particle interactions conserve some lepton charge, e.g., the total lepton charge L. If there does not exist any conserved lepton charge, neutrinos with definite mass can be Majorana particles. As is well-known, the massive neutrinos are predicted to be of Majorana nature by the see-saw mechanism of neutrino mass generation (see, e.g., [61]), which also provides a very attractive explanation of the smallness of the neutrino masses and - through the leptogenesis theory [57], of the observed baryon asymmetry of the Universe.

If the massive neutrinos ν_j are Majorana fermions, processes in which the total lepton charge L is not conserved and changes by two units, such as $K^+ \to \pi^- + \mu^+ + \mu^+$, $\mu^+ + (A, Z) \to (A, Z + 2) + \mu^-$, etc., should exist. The process most sensitive to the possible Majorana nature of neutrinos ν_j is the $(\beta\beta)_{0\nu}-$decay (see, e.g., [13, 62]). If the $(\beta\beta)_{0\nu}$-decay is generated *only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos ν_j ($m_j \lesssim 1$ eV)*, which will be assumed in what follows, the dependence of the $(\beta\beta)_{0\nu}$-decay amplitude $A(\beta\beta)_{0\nu}$ on the neutrino mass and mixing parameters factorizes in the effective Majorana mass $<m>$ (see, e.g., [13, 59]):

$$A(\beta\beta)_{0\nu} \sim <m> M \ , \tag{9}$$

where M is the corresponding nuclear matrix element (NME) and

$$|<m>| = \left| m_1 |U_{\mathrm{e}1}|^2 + m_2 |U_{\mathrm{e}2}|^2 \ e^{i\alpha_{21}} + m_3 |U_{\mathrm{e}3}|^2 \ e^{i\alpha_{31}} \right| \ . \tag{10}$$

If CP-invariance holds [7], one has [63] $\alpha_{21} = k\pi$, $\alpha_{31} = k'\pi$, where $k, k' = 0, 1, 2, ...$, and

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1 \tag{11}$$

represent the relative CP-parities of Majorana neutrinos ν_1 and ν_2, and ν_1 and ν_3, respectively. It follows from eq. (10) that the measurement of $|<m>|$ will provide information, in particular, on m_j. As eq. (9) indicates, the observation of $(\beta\beta)_{0\nu}$-decay of a given nucleus

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[7]We assume that $m_j > 0$ and that the fields of the Majorana neutrinos ν_j satisfy the Majorana condition: $C(\bar{\nu}_j)^T = \nu_j$, $j = 1, 2, 3$, where C is the charge conjugation matrix.

and the measurement of the corresponding half-life, would allow to determine $|<m>|$ only if the value of the relevant NME is known with a relatively small uncertainty.

The experimental searches for $(\beta\beta)_{0\nu}$−decay have a long history (see, e.g., [58, 59]). The best sensitivity was achieved in the Heidelberg-Moscow ^{76}Ge experiment [64]:

$$|<m>| < (0.35 - 1.05) \text{ eV}, \quad 90\% \text{ C.L.} \tag{12}$$

where a factor of 3 uncertainty associated with the calculation of the relevant nuclear matrix element [59] is taken into account. A positive signal at $>3\sigma$, corresponding to $|<m>| = (0.1 - 0.9)$ eV at 99.73% C.L., is claimed to be observed in [65]. This result will be checked in the currently running and future $(\beta\beta)_{0\nu}$-decay experiments. However, it may take a long time before comprehensive checks could be completed. Two experiments, NEMO3 (with ^{100}Mo and ^{82}Se) [66] and CUORICINO (with ^{130}Te) [67], designed to reach sensitivity to $|<m>| \sim$(0.2-0.3) eV, are taking data. Their first results read, respectively (90% C.L.):

$$|<m>| < (0.7 - 1.2) \text{ eV } [66], \quad |<m>| < (0.3 - 1.6) \text{ eV } [67], \tag{13}$$

where estimated uncertainties in the NME are accounted for. A number of projects aim to reach sensitivity to $|<m>| \sim$(0.01–0.05) eV [67, 62]: CUORE (^{130}Te), GENIUS (^{76}Ge), EXO (^{136}Xe), MAJORANA (^{76}Ge), MOON (^{100}Mo), XMASS (^{136}Xe), etc. These experiments can probe the region of IH and QD spectra and test the positive result claimed in [65].

The $(\beta\beta)_{0\nu}$-decay experiments are presently the only feasible experiments capable of establishing the Majorana nature of massive neutrinos [13, 59, 62]. As we will discuss in what follows, a measurement of a nonzero value of $|<m>| \gtrsim 10^{-2}$ eV:
– Can give also information on neutrino mass spectrum [38, 68, 69] (see also [70]).
– Can provide unique information on the absolute scale of neutrino masses (see, e.g., [68]).
– With additional information from other sources (^{3}H β-decay experiments and/or cosmological/astrophysical data) on the absolute ν-mass scale, can provide unique information on the Majorana CPV phases α_{21} and/or α_{31} [28, 37, 55, 68, 71].

3 Properties of Majorana Neutrinos: Brief Summary

The properties of Majorana fields (particles) are very different from those of Dirac fields (particles). A massive Majorana neutrino χ_k can be described (in local quantum field theory) by 4-component complex spin 1/2 field $\chi_k(x)$ which satisfies the Majorana condition:

$$C \ (\bar{\chi}_k(x))^{\mathrm{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1. \tag{14}$$

where C is the charge conjugation matrix. The Majorana condition is invariant under *proper* Lorentz transformations. It reduces by 2 the number of independent components in $\chi_k(x)$.

The condition (14) is invariant with respect to $U(1)$ global gauge transformations of the field $\chi_k(x)$ carrying a $U(1)$ charge Q, $\chi_k(x) \rightarrow e^{i\alpha Q} \chi_k(x)$, only if $Q = 0$. As a result, i) χ_k cannot carry nonzero additive quantum numbers (lepton charge, etc.), and ii) the field $\chi_k(x)$ cannot "absorb" phases [8]. Thus, $\chi_k(x)$ describes 2 spin states of a spin 1/2, *absolutely neutral particle*, which is identical with its antiparticle, $\chi_k \equiv \bar{\chi}_k$. If CP-invariance holds, Majorana neutrinos have definite CP-parity $\eta_{CP}(\chi_k) = \pm i$:

$$U_{CP} \ \chi_k(x) \ U_{CP}^{-1} = \eta_{CP}(\chi_k) \ \gamma_0 \ \chi_k(x'), \quad \eta_{CP}(\chi_k) = \pm i \ . \tag{15}$$

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[8]This is the reason why the $PMNS$ matrix contains two additional CPV phases in the case of massive Majorana neutrinos [26].

It follows from the Majorana condition that the currents $\bar{\chi}_k(x)O^i\chi_k(x) \equiv 0$, for $O^i = \gamma_\alpha$; $\sigma_{\alpha\beta}$; $\sigma_{\alpha\beta}\gamma_5$. This means that Majorana fermions (neutrinos) cannot have nonzero $U(1)$ charges and intrinsic magnetic and electric dipole moments.

Finally, if $\Psi(x)$ is a Dirac field and we define the standard propagator of $\Psi(x)$ as

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x - y) , \tag{16}$$

one has

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0 , \quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0 . \tag{17}$$

In contrast, a Majorana neutrino field $\chi_k(x)$ has, in addition to the standard propagator

$$< 0|T(\chi_{k\alpha}(x)\bar{\chi}_{k\beta}(y))|0 >= S^{Fk}_{\alpha\beta}(x - y) , \tag{18}$$

two non-trivial *non-standard (Majorana)* propagators [9]

$$< 0|T(\chi_{k\alpha}(x)\chi_{k\beta}(y))|0 >= -\xi^* S^{Fk}_{\alpha\delta}(x - y)C_{\delta\beta} ,$$
$$< 0|T(\bar{\chi}_{k\alpha}(x)\bar{\chi}_{k\beta}(y))|0 >= \xi \, C^{-1}_{\alpha\delta}S^{Fk}_{\delta\beta}(x - y) . \tag{19}$$

This result implies that if $\nu_j(x)$ in eq. (2) are massive Majorana neutrinos, $(\beta\beta)_{0\nu}$-decay can proceed by exchange of virtual neutrinos ν_j since $< 0|T(\nu_{j\alpha}(x)\nu_{j\beta}(y))|0 >\neq 0$.

4 Predictions for the Effective Majorana Mass

The predicted value of $|<m>|$ depends in the case of $3-\nu$ mixing on [72] (see also [28, 70]): i) $\Delta m^2_A = \Delta m^2_{31(32)}$, ii) $\theta_\odot = \theta_{12}$ and $\Delta m^2_\odot = \Delta m^2_{21}$, iii) the lightest neutrino mass, $min(m_j)$ and on iv) the mixing angle θ_{13}. In the convention (A) employed by us, one has $|U_{e1}|^2 = \cos^2\theta_\odot(1 - |U_{e3}|^2)$, $|U_{e2}|^2 = \sin^2\theta_\odot(1 - |U_{e3}|^2)$, and $|U_{e3}|^2 \equiv \sin^2\theta_{13}$.

Given $\Delta m^2_\odot$, Δm^2_A, $\theta_\odot$ and $\sin^2\theta_{13}$, the value of $|<m>|$ depends strongly on the type of the neutrino mass spectrum as well as on the values of the two Majorana CPV phases of the PMNS matrix, $\alpha_{21,31}$ (see eq. (10)). In the case of QD spectrum ($m_1 \cong m_2 \cong m_3 = m_0$, $m_0^2 \gg |\Delta m^2_A|, \Delta m^2_{21}$), $|<m>|$ is essentially independent on Δm^2_A and $\Delta m^2_\odot$, and the two possibilities, $\Delta m^2_A > 0$ and $\Delta m^2_A < 0$, lead *effectively* to the same predictions for $|<m>|$.

Normal Hierarchical Neutrino Mass Spectrum. In this case one has [28]

$$|<m>| = \left|(m_1\cos^2\theta_\odot + e^{i\alpha_{21}}\sqrt{\Delta m^2_\odot}\sin^2\theta_\odot)\cos^2\theta_{13} + \sqrt{\Delta m^2_A}\sin^2\theta_{13}\,e^{i\alpha_{31}}\right| \tag{20}$$

$$\simeq \left|\sqrt{\Delta m^2_\odot}\sin^2\theta_\odot\cos^2\theta_{13} + \sqrt{\Delta m^2_A}\sin^2\theta_{13}e^{i(\alpha_{31}-\alpha_{21})}\right| \tag{21}$$

where we have neglected the term $\sim m_1$ in eq. (21). Although neutrino ν_1 effectively "decouples" and does not contribute to $|<m>|$, eq. (21), the value of $|<m>|$ still depends on the Majorana CPV phase $\alpha_{32} = \alpha_{31} - \alpha_{21}$. This reflects the fact that in contrast to the case of massive Dirac neutrinos (or quarks), CP-violation can take place in the mixing of only two massive Majorana neutrinos [26]. Further, since [22] $\sqrt{\Delta m^2_\odot} \lesssim 9.5 \times 10^{-3}$ eV, $\sin^2\theta_\odot \lesssim 0.36$, $\sqrt{\Delta m^2_A} \lesssim 5.4 \times 10^{-2}$ eV [24] (at 90% C.L.), and the largest neutrino mass enters into the expression for $|<m>|$ with the factor $\sin^2\theta_{13} < 0.055$, the predicted value of $|<m>|$ is typically $\sim few \times 10^{-3}$ eV: for $\sin^2\theta_{13} = 0.04$ (0.02) one finds $|<m>| \lesssim 0.005$ (0.004) eV. Using the best fit values of the indicated parameters (see eqs. (1) and (6)) we get

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[9]For further detailed discussion of the properties of Majorana neutrinos (fermions) see, e.g., [13].

$|<m>| \lesssim 0.0044$ (0.0035) eV. It follows from eq. (20) and the allowed ranges of values of $\Delta m_\odot^2$, Δm_A^2, $\sin^2 \theta_\odot$, $\sin^2 \theta_{13}$ as well as of the lightest neutrino mass m_1 and CPV phases $\alpha_{21,31}$ that in the case of spectrum with *normal hierarchy* there can be a complete cancellation between the three terms in eq. (20) and one can have [68] $|<m>| = 0$.

Inverted Hierarchical Spectrum. For IH neutrino mass spectrum (see, e.g., [28]) $m_3 \ll m_1 \cong m_2 \cong \sqrt{|\Delta m_A^2|} = \sqrt{\Delta m_{23}^2}$. Neglecting $m_3 \sin^2 \theta_{13}$ in eq. (10), we find [37]:

$$|<m>| \cong \sqrt{|\Delta m_A^2|} \cos^2 \theta_{13} \sqrt{1 - \sin^2 2\theta_\odot \sin^2 \frac{\alpha_{21}}{2}}. \tag{22}$$

Even though one of the three massive Majorana neutrinos "decouples", the value of $|<m>|$ depends on the Majorana CP-violating phase α_{21}. Obviously,

$$\sqrt{|\Delta m_A^2|} \, |\cos 2\theta_\odot| \, \cos^2 \theta_{13} \leq \, |<m>| \leq \sqrt{|\Delta m_A^2|} \cos^2 \theta_{13}. \tag{23}$$

The upper and the lower limits correspond respectively to the CP-conserving cases. Most remarkably, since according to the solar neutrino and KamLAND data $\cos 2\theta_\odot \sim 0.40$, we get a significant lower limit on $|<m>|$, typically exceeding 10^{-2} eV, in this case [38, 68]. Using, e.g., the best fit values of $|\Delta m_A^2|$ and $\sin^2 \theta_\odot$ one finds: $|<m>| \gtrsim 0.02$ eV. The maximal value of $|<m>|$ is determined by $|\Delta m_A^2|$ and can reach, as it follows from eqs. (1) and (5), $|<m>| \sim 0.060$ eV. The indicated values of $|<m>|$ are within the range of sensitivity of the next generation of $(\beta\beta)_{0\nu}$-decay experiments.

The expression for $|<m>|$, eq. (22), permits to relate the value of $\sin^2 \alpha_{21}/2$ to the experimentally measured quantities [28, 37] $|<m>|$, Δm_A^2 and $\sin^2 2\theta_\odot$:

$$\sin^2 \frac{\alpha_{21}}{2} \cong \left(1 - \frac{|<m>|^2}{|\Delta m_A^2| \cos^4 \theta_{13}}\right) \frac{1}{\sin^2 2\theta_\odot}. \tag{24}$$

A sufficiently accurate measurement of $|<m>|$ and of $|\Delta m_A^2|$ and $\theta_\odot$, could allow to get information about the value of α_{21}, provided the neutrino mass spectrum is of the IH type.

Three Quasi-Degenerate Neutrinos. In this case $m_0 \equiv m_1 \cong m_2 \cong m_3$, $m_0^2 \gg |\Delta m_A^2|$, $m_0 \gtrsim 0.20$ eV. The mass m_0 effectively coincides with the $\bar{\nu}_e$ mass $m_{\bar{\nu}_e}$ measured in the ^{3}H β-decay experiments: $m_0 = m_{\bar{\nu}_e}$. Thus, $m_0 < 2.3$ eV, or if we use a conservative cosmological upper limit [42], $m_0 < 0.7$ eV. The QD ν-mass spectrum is realized for values of m_0, which can be measured in the ^{3}H $\beta-$decay experiment KATRIN [41].

The effective Majorana mass $|<m>|$ is given by

$$|<m>| \cong m_0 \left|(\cos^2 \theta_\odot + \sin^2 \theta_\odot e^{i\alpha_{21}}) \cos^2 \theta_{13} + e^{i\alpha_{31}} \sin^2 \theta_{13}\right| \tag{25}$$

$$\cong m_0 \left|\cos^2 \theta_\odot + \sin^2 \theta_\odot e^{i\alpha_{21}}\right| = m_0 \sqrt{1 - \sin^2 2\theta_\odot \sin^2 \frac{\alpha_{21}}{2}}. \tag{26}$$

Similarly to the case of IH spectrum, one has:

$$m_0 \, |\cos 2\theta_\odot| \, \lesssim |<m>| \lesssim m_0. \tag{27}$$

For $\cos 2\theta_\odot \sim 0.40$, favored by the data, one finds a non-trivial lower limit on $|<m>|$, $|<m>| \gtrsim 0.08$ eV. For the 90% C.L. allowed ranges of values of the parameters one has $|<m>| \gtrsim 0.06$ eV. Using the conservative cosmological upper bound on $\sum_j m_j$ we get $|<m>| < 0.70$ eV. Also in this case one can obtain, in principle, a direct information on one
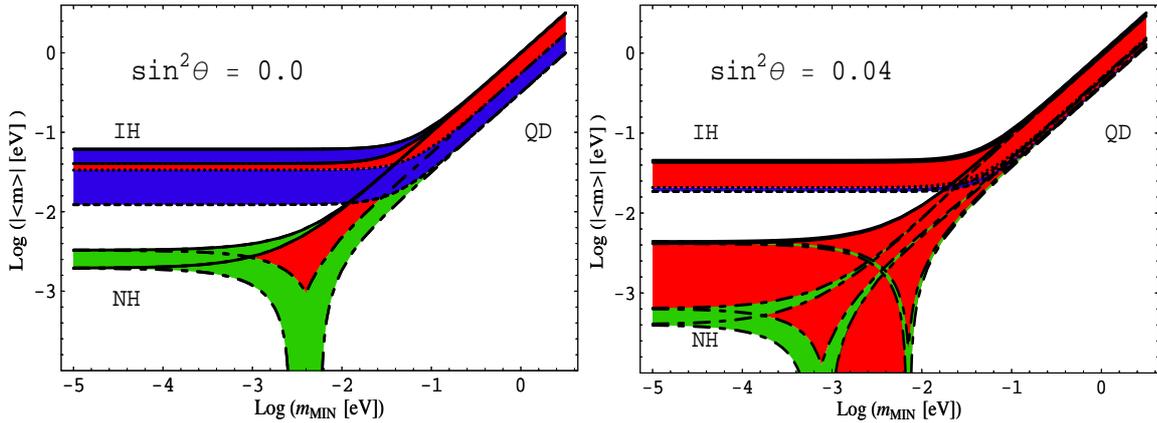
Figure 2: The value of $|<m>|$ as function of $min(m_j)$ for $\sin^2\theta_{13}$=0.0;0.04 and 90% C.L. allowed ranges [22] of Δm_A^2 , $\Delta m_\odot^2$ and $\theta_\odot$ (updated version of Fig. 2 from [38]).

CPV phase from the measurement of $|<m>|$, m_0 and $\sin^2 2\theta_\odot$:

$$\sin^2 \frac{\alpha_{21}}{2} \cong \left(1 - \frac{|<m>|^2}{m_0^2}\right) \frac{1}{\sin^2 2\theta_\odot}. \tag{28}$$

The specific features of the predictions for $|<m>|$ in the cases of the three types of neutrino mass spectrum discussed above are evident in Fig. 2, where the dependence of $|<m>|$ on $min(m_j)$ for the LMA solution is shown. If the spectrum is with normal hierarchy, $|<m>|$ can lie anywhere between 0 and the presently existing upper limits, eqs. (12) and (13). This conclusion does not change even under the most favorable conditions for the determination of $|<m>|$, namely, even when $|\Delta m_A^2|$, $\Delta m_\odot^2$, $\theta_\odot$ and θ_{13} are known with negligible uncertainty. If the results in [65] implying $|<m>| = (0.1 - 0.9)$ eV are confirmed, this would mean, in particular, that the neutrino mass spectrum is of the QD type.

5   Implications of Measuring $|<m>| \neq 0$

If the $(\beta\beta)_{0\nu}$-decay of a given nucleus will be observed, it would be possible to determine the value of $|<m>|$ from the measurement of the associated half-life of the decay. This would require the knowledge of the nuclear matrix element of the process.

On the NME Uncertainties. At present there exist large uncertainties in the calculation of the $(\beta\beta)_{0\nu}$-decay nuclear matrix elements (see, e.g., [59]). This is reflected, in particular, in the factor of ~ 3 uncertainty in the upper limit on $|<m>|$, which is extracted from the experimental lower limits on the $(\beta\beta)_{0\nu}$-decay half-life of ^{76}Ge. Recently, encouraging results in what regards the problem of the calculation of the nuclear matrix elements have been obtained in [73]. The observation of a $(\beta\beta)_{0\nu}$-decay of one nucleus is likely to lead to the searches and eventually to observation of the decay of other nuclei. One can expect that such a progress, in particular, will help to solve the problem of the sufficiently precise calculation of the nuclear matrix elements for the $(\beta\beta)_{0\nu}$-decay [74].

Constraining the Lightest Neutrino Mass. As Fig. 2 indicates, a measurement of $|<m>| \gtrsim 0.01$ eV would either i) determine a relatively narrow interval of possible values of the lightest ν-mass $min(m_j) \equiv m_{\text{MIN}}$, or ii) would establish an upper limit on m_{MIN}. If an upper limit on $|<m>|$ is experimentally obtained below 0.01 eV, this would lead to a significant upper limit on m_{MIN} and would imply $\Delta m_A^2 > 0$ for massive Majorana neutrinos.

A measurement of $|<m>| = (|<m>|)_{exp} \gtrsim 0.02$ eV if $\Delta m_{\rm A}^2 \equiv \Delta m_{31}^2 > 0$, and of $|<m>| = (|<m>|)_{exp} \gtrsim \sqrt{|\Delta m_{\rm A}^2|} \cos^2 \theta_{13}$ in the case of $\Delta m_{\rm A}^2 \equiv \Delta m_{32}^2 < 0$, for instance, would imply that $m_{\rm MIN} \gtrsim 0.02$ eV and $m_{\rm MIN} \gtrsim 0.05$ eV, respectively, and thus a ν-mass spectrum with *partial hierarchy* or of QD type [28]. The mass $m_{\rm MIN}$ will be constrained to lie in a rather narrow interval [68] (Fig. 2). If the measured value of $|<m>|$, $(|<m>|)_{exp}$, lies between the $min(|<m>|)$ and $max(|<m>|)$, predicted in the case of IH spectrum,

$$|<m>|_{\pm} = \left| \sqrt{|\Delta m_{\rm A}^2| - \Delta m_{\odot}^2} \cos^2 \theta_{\odot} \pm \sqrt{|\Delta m_{\rm A}^2|} \sin^2 \theta_{\odot} \right| \cos^2 \theta, \qquad (29)$$

$m_{\rm MIN}$ would be limited from above, but $min(m_{\rm MIN})=0$ (Fig. 2). If $(|<m>|)_{exp} < (|<m>|)_{max}$, where, e.g., in the case of QD spectrum $(|<m>|)_{max} \cong m_{\rm MIN} \cong m_{\bar{\nu}_e}$, this would imply that at least one of the two CPV phases is different from zero : $\alpha_{21} \neq 0$ and/or $\alpha_{31} \neq 0$ [10].

A measured value of $m_{\bar{\nu}_e}$, $(m_{\bar{\nu}_e})_{exp} \gtrsim 0.20$ eV, satisfying $(m_{\bar{\nu}_e})_{exp} > (m_{\rm MIN})_{max}$, where $(m_{\rm MIN})_{max}$ is determined from the upper limit on $|<m>|$ in the case the $(\beta\beta)_{0\nu}$-decay is not observed, might imply that the massive neutrinos are Dirac particles. If $(\beta\beta)_{0\nu}$-decay has been observed and $|<m>|$ measured, the inequality $(m_{\bar{\nu}_e})_{exp} > (m_{\rm MIN})_{max}$, with $(m_{\rm MIN})_{max}$ determined from the measured value of $|<m>|$, would lead to the conclusion that there exist contribution(s) to the $(\beta\beta)_{0\nu}$-decay rate other than due to the light Majorana neutrino exchange that partially cancels the contribution from the Majorana neutrino exchange.

Determining the Type of Neutrino Mass Spectrum. The existence of significant lower bounds on $|<m>|$ in the cases of IH and QD spectra [38], which lie either partially (IH spectrum) or completely (QD spectrum) within the range of sensitivity of next generation of $(\beta\beta)_{0\nu}$-decay experiments, is one of the most important features of the predictions of $|<m>|$. These minimal values are given, up to small corrections, by $\sqrt{|\Delta m_{\rm A}^2|} \cos 2\theta_{\odot}$ and $m_0 \cos 2\theta_{\odot}$. According to the combined analysis of the solar and reactor neutrino data [22] i) $\cos 2\theta_{\odot} = 0$ is excluded at $\sim 6\sigma$, ii) the best fit value of $\cos 2\theta_{\odot}$ is $\cos 2\theta_{\odot} = 0.44$, and iii) at 95% C.L. one has for $\sin^2 \theta_{13} = 0$ (0.02), $\cos 2\theta_{\odot} \gtrsim 0.28$ (0.30). The quoted results on $\cos 2\theta_{\odot}$ together with the range of possible values of $|\Delta m_{\rm A}^2|$ and m_0 [3, 24, 41, 42], lead to the conclusion about the existence of significant and robust lower bounds on $|<m>|$ in the cases of IH and QD spectrum [38, 75]. At the same time, as Fig. 2 indicates, $|<m>|$ does not exceed ~ 0.006 eV for NH spectrum. This implies that $max(|<m>|)$ in the case of NH spectrum is considerably smaller than $min(|<m>|)$ for the IH and QD spectrum. This opens the possibility of obtaining information about the type of ν-mass spectrum from a measurement of $|<m>| \neq 0$. In particular, a positive result in the future generation of $(\beta\beta)_{0\nu}$-decay experiments showing that $|<m>| > 0.02$ eV would imply that the NH spectrum is strongly disfavored (if not excluded). The uncertainty in the relevant NME and prospective experimental errors in the values of the oscillation parameters and in $|<m>|$ can weaken, but do not invalidate, these results (see, e.g., ref. [69]).

Constraining the Majorana CPV Phases. The possibility of establishing CP-violation due to Majorana CPV phases has been studied in [68] and in greater detail in [55]. It was found that it is very challenging [11]: it requires quite accurate measurements

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[10]In general, the knowledge of the value of $|<m>|$ alone will not allow to distinguish the case of CP-conservation from that of CP-violation.

[11]Pessimistic conclusion about the prospects to establish CP-violation due to Majorana CPV phases from a measurement of $|<m>|$ and, e.g., of m_0, was reached in [53].

of $|<m>|$ and of m_0, and holds only for a limited range of values of the relevant parameters. For IH and QD spectra, which are of interest, the "just CP-violation" region [28] - an experimental point in this region would signal unambiguously CP-violation associated with Majorana neutrinos, is larger for smaller values of $\cos 2\theta_\odot$. More specifically, proving that CP-violation associated with Majorana neutrinos takes place requires, in particular, a relative experimental error on the measured value of $|<m>|$ smaller than $\sim 15\%$, a "theoretical uncertainty" in the value of $|<m>|$ due to an imprecise knowledge of the corresponding NME smaller than a factor of 2, a value of $\tan^2 \theta_\odot \gtrsim 0.55$, and values of the relevant Majorana CPV phases typically within the ranges of $\sim (\pi/2 - 3\pi/4)$ and $\sim (5\pi/4 - 3\pi/2)$.

6   Conclusions

Future $(\beta\beta)_{0\nu}$−decay experiments have a remarkable physics potential. They can establish the Majorana nature of the neutrinos with definite mass ν_j. If the latter are Majorana particles, the $(\beta\beta)_{0\nu}$−decay experiments can provide information on the type of the neutrino mass spectrum and on the absolute scale of neutrino masses. They can also provide unique information on the Majorana CP-violation phases present in the PMNS neutrino mixing matrix. The knowledge of the values of the relevant $(\beta\beta)_{0\nu}$−decay nuclear matrix elements with a sufficiently small uncertainty is crucial for obtaining quantitative information on the neutrino mass and mixing parameters from a measurement of $(\beta\beta)_{0\nu}$−decay half-life.

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