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# The See-Saw Mechanism, Neutrino Yukawa Couplings, LFV Decays $l_i \to l_j + \gamma$ and Leptogenesis


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## Abstract

The LFV charged lepton decays $\mu \to e+\gamma$, $\tau \to e+\gamma$ and $\tau \to \mu+\gamma$ and thermal leptogenesis are analysed in the MSSM with see-saw mechanism of neutrino mass generation and soft SUSY breaking with universal boundary conditions. The case of hierarchical heavy Majorana neutrino mass spectrum, $M_1 \ll M_2 \ll M_3$, is investigated. Leptogenesis requires $M_1 \gtrsim 10^9$ GeV. Considering the natural range of values of the heaviest right-handed Majorana neutrino mass, $M_3 \gtrsim 5 \times 10^{13}$ GeV, and assuming that the soft SUSY breaking universal gaugino and/or scalar masses have values in the range of few $\times$ 100 GeV, we derive the combined constraints, which the existing stringent upper limit on the $\mu \to e + \gamma$ decay rate and the requirement of successful thermal leptogenesis impose on the neutrino Yukawa couplings, heavy Majorana neutrino masses and SUSY parameters. Results for the three possible types of light neutrino mass spectrum – normal and inverted hierarchical and quasi-degenerate – are obtained.


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# 1 Introduction

The experiments with solar, atmospheric, reactor and accelerator neutrinos [1, 2, 3, 4, 5] have provided during the last several years compelling evidence for the existence of non-trivial 3-neutrino mixing in the weak charged-lepton current (see, *e.g.*, [6]):

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}}, \quad l = e, \mu, \tau, \tag{1}$$

where $\nu_{lL}$ are the flavour neutrino fields, $\nu_{j\mathrm{L}}$ is the field of neutrino $\nu_j$ having a mass $m_j$ and $U$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [7], $U \equiv U_{\mathrm{PMNS}}$. The existing data, including the data from the $^3$H $\beta$-decay experiments [8] imply that the massive neutrinos $\nu_j$ are significantly lighter than the charged leptons and quarks: $m_j < 2.3$ eV (95% C.L.) [1]. A natural explanation of the smallness of neutrino masses is provided by the see-saw mechanism of neutrino mass generation [11]. The see-saw mechanism predicts the light massive neutrinos $\nu_j$ to be Majorana particles. An integral part of the mechanism are the heavy right-handed (RH) Majorana neutrinos [12]. In grand unified theories (GUT) the masses of the heavy RH Majorana neutrinos are typically by a few to several orders of magnitude smaller than the scale of unification of the electroweak and strong interactions, $M_{\mathrm{GUT}} \cong 2 \times 10^{16}$ GeV. In this case the CP-violating decays of the heavy RH Majorana neutrinos in the Early Universe could generate, through the leptogenesis scenario, the observed baryon asymmetry of the Universe [13].

The existence of the flavour neutrino mixing, eq. (1), implies that the individual lepton charges, $L_l$, $l = e, \mu, \tau$, are not conserved (see, *e.g.*, [14]), and processes like $\mu^- \to e^- + \gamma$, $\mu^- \to e^- + e^+ + e^-$, $\tau^- \to e^- + \gamma$, $\tau^- \to \mu^- + \gamma$, $\mu^- + (A, Z) \to e^- + (A, Z)$, *etc.* should take place. Stringent experimental upper limits on the branching ratios and relative cross sections of the indicated $|\Delta L_l| = 1$ decays and reactions have been obtained [15, 16, 17] (90% C.L.):

$$\begin{gathered} \mathrm{B}(\mu \to e + \gamma) < 1.2 \times 10^{-11}, \quad \mathrm{B}(\mu \to 3e) < 1.2 \times 10^{-12}, \quad \mathrm{R}(\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}) < 4.3 \times 10^{-12}\ , \\ \mathrm{B}(\tau \to \mu + \gamma) < 6.8 \times 10^{-8}\ , \quad \mathrm{B}(\tau \to e + \gamma) < 1.1 \times 10^{-7}\ . \end{gathered} \tag{2}$$

Future experiments with increased sensitivity can reduce the current bounds on $\mathrm{B}(\mu \to e + \gamma)$, $\mathrm{B}(\tau \to \mu + \gamma)$ and on $\mathrm{R}(\mu^- + (A, Z) \to e^- + (A, Z))$ by a few orders of magnitude (see, *e.g.*, [18]). In the experiment MEG under preparation at PSI [19] it is planned to reach a sensitivity to

$$\mathrm{B}(\mu \to e + \gamma) \sim (10^{-13} - 10^{-14})\,. \tag{3}$$

It has been noticed a long time ago that in SUSY (GUT) theories with see-saw mechanism of neutrino mass generation, the rates and cross sections of the LFV processes can be strongly enhanced [20]. If the SUSY breaking occurs via soft terms with universal boundary conditions at a scale $M_X$ above the RH Majorana neutrino mass scale $M_R$, $M_X > M_R$ [2], the renormalisation group (RG) effects transmit the LFV from the neutrino mixing at $M_X$ to the effective mass terms of the scalar leptons at $M_R$ even if the soft SUSY breaking terms at $M_X$ are flavour symmetric and conserve the lepton charges $L_l$. As a consequence of these RG-induced new LFV terms in the

[1] More stringent upper limit on $m_j$ follows from the constraints on the sum of neutrino masses obtained from cosmological/astrophysical observations, namely, the CMB data of the WMAP experiment combined with data from large scale structure surveys (2dFGRS, SDSS) [9]: $\sum_j m_j < (0.7 - 2.0)$ eV (95% C.L.), where we have included a conservative estimate of the uncertainty in the upper limit (see, *e.g.*, [10]).

[2] The possibility of "flavour-blind" SUSY breaking of interest is realised, *e.g.*, in gravity-mediation SUSY breaking scenarios (see, *e.g.*, [21]).

effective Lagrangian at $M_R < M_X$, the LFV processes can proceed with rates and cross sections which are within the sensitivity of presently operating and future planned experiments [20, 22] (see also, *e.g.*, [23, 24, 25, 26, 27, 28, 29, 30]). In contrast, in the non-supersymmetric case, the rates and cross sections of the LFV processes are suppressed by the factor [31] (see also [32]) $(m_j/M_W)^4 < 6.7 \times 10^{-43}$, $M_W$ being the $W^\pm$ mass, which renders them unobservable.

One of the basic ingredients of the see-saw mechanism is the matrix of neutrino Yukawa couplings, $\mathbf{Y}_\nu$. Leptogenesis depends on $\mathbf{Y}_\nu$ as well [13] (see also [33, 34] and the references quoted therein). In the large class of SUSY models with see-saw mechanism and SUSY breaking mediated by flavour-universal soft terms at a scale $M_X > M_R$ we will consider, the probabilities of LFV processes also depend strongly on $\mathbf{Y}_\nu$ (see, *e.g.*, [23, 24]). The matrix $\mathbf{Y}_\nu$ can be expressed in terms of the light neutrino and heavy RH neutrino masses, the neutrino mixing matrix $U_{\rm PMNS}$, and an orthogonal matrix $\mathbf{R}$ [23]. Leptogenesis can take place only if $\mathbf{R}$ is complex. The matrix $\mathbf{Y}_\nu$ depends, in particular, on the Majorana CP-violation (CPV) phases in the PMNS matrix $U_{\rm PMNS}$ [35] [3]. It was shown in [26, 30] that if the heavy Majorana neutrinos are quasi-degenerate in mass, the Majorana phases can affect significantly the predictions for the rates of LFV decays $\mu \to e + \gamma$, $\tau \to e + \gamma$, *etc.* in the class of SUSY theories of interest.

The matrix $\mathbf{Y}_\nu$ can be defined, strictly speaking, only at scales not smaller than $M_R$. The probabilities of LFV processes depend on $\mathbf{Y}_\nu$ at the scale $M_R$, $\mathbf{Y}_\nu = \mathbf{Y}_\nu(M_R)$. In order to evaluate $\mathbf{Y}_\nu(M_R)$ one has to know, in general, the light neutrino masses $m_j$ and the mixing matrix $U_{\rm PMNS}$ at $M_R$, *i.e.*, one has to take into account the renormalisation group (RG) "running" of $m_j$ and $U_{\rm PMNS}$ from the scale $M_Z \sim 100$ GeV, at which the neutrino mixing parameters are measured, to the scale $M_R$ (see, *e.g.*, [40, 30] and the references quoted therein). However, if the RG running of $m_j$ and $U_{\rm PMNS}$ is sufficiently small, $\mathbf{Y}_\nu(M_R)$ will depend on the values of the light neutrino masses $m_j$ and the mixing angles and CP-violation phases in $U_{\rm PMNS}$ at the scale $M_Z$.

Working in the framework of the class of SUSY theories with see-saw mechanism and soft SUSY breaking with flavour-universal boundary conditions at a scale $M_X > M_R$, we investigate in the present article the combined constraints, which the existing stringent upper limit of the $\mu \to e + \gamma$ decay rate and the requirement of successful thermal leptogenesis impose on the neutrino Yukawa couplings, heavy Majorana neutrino masses and on the SUSY parameters. The case of hierarchical heavy Majorana neutrino mass spectrum, $M_1 \ll M_2 \ll M_3$, is considered. Leptogenesis requires $M_1 \gtrsim 10^9$ GeV. The analysis is performed assuming that the heaviest RH Majorana neutrino has a mass $M_3 \gtrsim 5 \times 10^{13}$ GeV, and that the soft SUSY breaking universal gaugino and/or scalar masses (at the scale $M_X$) have values in the range of few $\times$ 100 GeV. One typically gets $M_3 \gtrsim 5 \times 10^{13}$ GeV in SUSY GUT theories with see-saw mechanism of neutrino mass generation (see, *e.g.*, [41]). If the SUSY breaking universal gaugino and/or scalar masses have values in the few $\times$ 100 GeV range, supersymmetric particles will be observable in the experiments under preparation at the LHC (see, *e.g.*, [42]). We find that under the indicated assumptions, the existing stringent upper limit on the $\mu \to e + \gamma$ decay rate cannot be satisfied, unless the terms proportional to $M_3$ in the $\mu \to e + \gamma$ decay amplitude are absent or strongly suppressed. The requisite "decoupling" of the terms $\propto M_3$ from the $\mu \to e + \gamma$ decay amplitude is realised if the matrix $\mathbf{R}$ has a specific form which admits a parametrisation with just one complex angle. Using the latter we obtain results for the three types of light neutrino mass spectrum – normal and inverted hierarchical (NH and

[3] Obtaining information about the Majorana CPV phases in the PMNS matrix $U_{\rm PMNS}$ if the massive neutrinos are proved to be Majorana particles would be a remarkably challenging problem. The oscillations of flavour neutrinos, $\nu_l \to \nu_{l'}$ and $\bar{\nu}_l \to \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$, are insensitive to the two Majorana phases in $U_{\rm PMNS}$ [35, 36]. The only feasible experiments that at present have the potential of establishing the Majorana nature of light neutrinos $\nu_j$ and of providing information on the Majorana phases in $U_{\rm PMNS}$ are the experiments searching for neutrinoless double beta ($(\beta\beta)_{0\nu}$)-decay, $(A, Z) \to (A, Z + 2) + e^- + e^-$ (see, *e.g.*, [14, 37, 38, 39]).

IH), and quasi-degenerate (QD). For each of the three types of spectrum we derive the leptogenesis lower bound on the mass of the lightest RH Majorana neutrino $M_1$. The lower bounds thus found in the cases of IH and QD spectrum are $\sim 10^{13}$ GeV. The upper limit on $\mathrm{B}(\mu \to e+\gamma)$ in these two cases can be satisfied for specific ranges of values of the soft SUSY breaking parameters implying relatively large masses of the supersymmetric particles. Using these soft SUSY breaking parameters we derive predictions for $\mathrm{B}(\mu \to e+\gamma)$, $\mathrm{B}(\tau \to e+\gamma)$ and $\mathrm{B}(\tau \to \mu+\gamma)$ which are compatible with the requirement of successful leptogenesis.

Our analysis is performed under the condition of negligible RG effects for the light neutrino masses $m_j$ and the mixing angles and CP-violation phases in $U_{\mathrm{PMNS}}$. The RG effects in question are negligible in the class of SUSY theories we are considering in the case of hierarchical light neutrino mass spectrum (see, *e.g.*, [40, 30]). The same is valid for quasi-degenerate $\nu_j$ mass spectrum provided the parameter $\tan\beta < 10$, $\tan\beta$ being the ratio of the vacuum expectation values of the up- and down-type Higgs doublet fields in SUSY extensions of the Standard Theory.

## 2 General Considerations

### 2.1 Neutrino Mixing Parameters from Neutrino Oscillation Data

We will use the standard parametrisation of the PMNS matrix $U_{\mathrm{PMNS}}$ (see, *e.g.*, [38]):

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \mathrm{diag}(1, e^{i\frac{\alpha}{2}}, e^{i\frac{\beta_M}{2}})\,, \quad (4)$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, the angles $\theta_{ij} = [0, \pi/2]$, $\delta = [0, 2\pi]$ is the Dirac CP-violating phase and $\alpha$ and $\beta_M$ are two Majorana CP-violation phases [35, 43]. One can identify the neutrino mass squared difference responsible for solar neutrino oscillations, $\Delta m^2_{\odot}$, with $\Delta m^2_{21} \equiv m^2_2 - m^2_1$, $\Delta m^2_{\odot} = \Delta m^2_{21} > 0$. The neutrino mass squared difference driving the dominant $\nu_\mu \to \nu_\tau$ ($\bar\nu_\mu \to \bar\nu_\tau$) oscillations of atmospheric $\nu_\mu$ ($\bar\nu_\mu$) is then given by $\left|\Delta m^2_{\mathrm{A}}\right| = \left|\Delta m^2_{31}\right| \cong \left|\Delta m^2_{32}\right| \gg \Delta m^2_{21}$. The corresponding solar and atmospheric neutrino mixing angles, $\theta_\odot$ and $\theta_{\mathrm{A}}$, coincide with $\theta_{12}$ and $\theta_{23}$, respectively. The angle $\theta_{13}$ is limited by the data from the CHOOZ and Palo Verde experiments [44].

The existing neutrino oscillation data allow us to determine $\Delta m^2_{21}$, $\left|\Delta m^2_{31}\right|$, $\sin^2\theta_{12}$ and $\sin^2 2\theta_{23}$ with a relatively good precision and to obtain rather stringent limits on $\sin^2\theta_{13}$ (see, *e.g.*, [2, 45, 46]). The best fit values of $\Delta m^2_{21}$, $\sin^2\theta_{12}$, $\left|\Delta m^2_{31}\right|$ and $\sin^2 2\theta_{23}$ read [4]:

$$\Delta m^2_{21} = 8.0 \times 10^{-5}\ \mathrm{eV}^2, \quad \sin^2\theta_{12} = 0.31\ , \quad (5)$$

$$\left|\Delta m^2_{31}\right| = 2.1 \times 10^{-3}\ \mathrm{eV}^2\ , \quad \sin^2 2\theta_{23} = 1.0\ , \quad (6)$$

A combined 3-$\nu$ oscillation analysis of the solar neutrino, KamLAND and CHOOZ data gives [45]

$$\sin^2\theta_{13} < 0.024\ (0.044), \qquad \text{at } 95\%\ (99.73\%)\ \text{C.L.} \quad (7)$$

The neutrino oscillation parameters $\Delta m^2_{21}$, $\sin^2\theta_{12}$, $|\Delta m^2_{31}|$ and $\sin^2 2\theta_{23}$ are determined by the existing data at $3\sigma$ with an error of approximately 12%, 24%, 50% and 16%, respectively. These parameters can (and very likely will) be measured with much higher accuracy in the future (see,

[4]The data imply, in particular, that maximal solar neutrino mixing is ruled out at $\sim 6\sigma$; at 95% C.L. one finds $\cos 2\theta_\odot \geq 0.26$ [45], which has important implications [47].

*e.g.*, [6]). In all further numerical estimates we use the best fit values of $\Delta m^2_{21}$, $\sin^2\theta_{12}$, $|\Delta m^2_{31}|$ and $\sin^2 2\theta_{23}$. Whenever the parameter $\sin\theta_{13}$ is also relevant in the calculations, we specify the value used.

The sign of $\Delta m^2_{\rm A} = \Delta m^2_{31}$, as it is well known, cannot be determined from the present (SK atmospheric neutrino and K2K) data. The two possibilities, $\Delta m^2_{31(32)} > 0$ or $\Delta m^2_{31(32)} < 0$ correspond to two different types of $\nu$-mass spectrum:
– *with normal hierarchy* $m_1 < m_2 < m_3$, $\Delta m^2_{\rm A} = \Delta m^2_{31} > 0$, and
– *with inverted hierarchy* $m_3 < m_1 < m_2$, $\Delta m^2_{\rm A} = \Delta m^2_{32} < 0$.
Depending on the sign of $\Delta m^2_{\rm A}$, ${\rm sgn}(\Delta m^2_{\rm A})$, and the value of the lightest neutrino mass, $\min(m_j)$, the $\nu$-mass spectrum can be
– *Normal Hierarchical*: $m_1 \ll m_2 \ll m_3$, $m_2 \cong (\Delta m^2_{\odot})^{\frac{1}{2}} \sim 0.009$ eV, $m_3 \cong \left|\Delta m^2_{\rm A}\right|^{\frac{1}{2}} \sim 0.045$ eV;
– *Inverted Hierarchical*: $m_3 \ll m_1 < m_2$, with $m_{1,2} \cong \left|\Delta m^2_{\rm A}\right|^{\frac{1}{2}} \sim 0.045$ eV;
– *Quasi-Degenerate (QD)*: $m_1 \cong m_2 \cong m_3 \cong m$, $m_j^2 \gg \left|\Delta m^2_{\rm A}\right|$, $m \gtrsim 0.10$ eV.

### 2.2 The See-Saw Mechanism and Neutrino Yukawa Couplings

We consider the minimal supersymmetric standard model with RH neutrinos and see-saw mechanism of neutrino mass generation (MSSMRN). In the framework of MSSMRN one can always choose a basis in which both the matrix of charged lepton Yukawa couplings, $\mathbf{Y}_{\rm E}$, and the Majorana mass matrix of the heavy RH neutrinos, $\mathbf{M}_{\rm N}$, are real and diagonal. Henceforth, we will work in that basis and will denote by $\mathbf{D}_{\rm N}$ the corresponding diagonal RH neutrino mass matrix, $\mathbf{D}_{\rm N} = {\rm diag}(M_1, M_2, M_3)$, with $M_j > 0$ and $M_1 < M_2 < M_3$. The largest mass $M_3$ will be standardly assumed to be of the order of, or smaller than, the GUT scale $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV.

Below the see-saw scale, $M_R = \min(M_j)$, the heavy RH neutrino fields $N_j$ are integrated out, and as a result of the electroweak symmetry breaking, the left-handed (LH) flavour neutrinos acquire a Majorana mass term:

$$\mathcal{L}^{\nu}_m = -\frac{1}{2}\, \bar{\nu}^C_{Rj}\, (m_\nu)^{jk}\, \nu_{Lk} + h.c.\ , \tag{8}$$

where $\nu^C_{Rj} \equiv C(\bar{\nu}_{Lj})^T$ and

$$(m_\nu)^{ij} = v_u^2\, (Y_\nu^T)^{ik} (M_{\rm N}^{-1})^{kl} (Y_\nu)^{lj}\ . \tag{9}$$

Here $v_u = v\sin\beta$, where $v = 174$ GeV and $\tan\beta$ is the ratio of the vacuum expectation values of up-type and down-type Higgs fields, and $\mathbf{Y}_\nu$ is the matrix of neutrino Yukawa couplings. The neutrino mass matrix $\mathbf{m}_\nu$ is related to the light neutrino masses $m_j$ and the PMNS mixing matrix as follows

$$(m_\nu)^{ij} = (U^*)^{ik} m_k (U^\dagger)^{kj}\ . \tag{10}$$

Using (9) and (10), we can rewrite the "matching condition" at the energy scale $M_R$ in the form

$$\mathbf{U}^*\, \mathbf{D}_\nu\, \mathbf{U}^\dagger = v_u^2\, \mathbf{Y}_\nu^T \mathbf{M}_{\rm N}^{-1} \mathbf{Y}_\nu\ . \tag{11}$$

where $\mathbf{D}_\nu = {\rm diag}(m_1, m_2, m_3)$. Thus, in the basis in which the RH neutrino mass matrix is diagonal $\mathbf{M}_{\rm N} = \mathbf{D}_{\rm N}$, the matrix of neutrino Yukawa couplings at $M_R$ can be parametrised as [23]

$$\mathbf{Y}_\nu(M_R) = \frac{1}{v_u}\sqrt{\mathbf{D}_N}\ \mathbf{R}\ \sqrt{\mathbf{D}_\nu}\ \mathbf{U}^\dagger\ . \tag{12}$$

Here $\mathbf{R}$ is a complex orthogonal matrix [5] $\mathbf{R}^T\mathbf{R} = \mathbf{1}$.

In what follows we will investigate the case when the RG running of $m_j$ and of the parameters in $U_{\text{PMNS}}$ from $M_Z$ to $M_R$ is relatively small and can be neglected. This possibility is realised in the class of theories under discussion for sufficiently small values of $\tan\beta$ and/or of the lightest neutrino mass $\min(m_j)$ [30], *e.g.*, for $\tan\beta \lesssim 10$ and/or $\min(m_j) \lesssim 0.05$ eV. Under the indicated condition $\mathbf{D}_\nu$ and $\mathbf{U}$ in eq. (12) can be taken at the scale $\sim M_Z$, at which the neutrino mixing parameters are measured.

As is well-known and we shall discuss further, in the case of soft SUSY breaking mediated by soft flavour-universal terms at $M_X > M_R$, the predicted rates of LFV processes such as $\mu \to e + \gamma$ decay are very sensitive to the off-diagonal elements of

$$\mathbf{Y}_\nu^\dagger(M_R)\mathbf{Y}_\nu(M_R) = \frac{1}{v_u^2}\, \mathbf{U}\sqrt{\mathbf{D}_\nu}\, \mathbf{R}^\dagger\, \mathbf{D}_N\, \mathbf{R}\, \sqrt{\mathbf{D}_\nu}\mathbf{U}^\dagger\, , \tag{13}$$

while leptogenesis depends on [13] (see also [33, 34] and the references quoted therein)

$$\mathbf{Y}_\nu(M_R)\mathbf{Y}_\nu^\dagger(M_R) = \frac{1}{v_u^2}\, \sqrt{\mathbf{D}_\mathrm{N}}\, \mathbf{R}\, \mathbf{D}_\nu\, \mathbf{R}^\dagger\, \sqrt{\mathbf{D}_\mathrm{N}}\, . \tag{14}$$

In such a way, the matrix of neutrino Yukawa couplings $\mathbf{Y}_\nu$ connects in the see-saw theories the light neutrino mass generation with leptogenesis; in SUSY theories with SUSY breaking mediated by soft flavour-universal terms in the Lagrangian at $M_X > M_R$, $\mathbf{Y}_\nu$ links the light neutrino mass generation and leptogenesis with LFV processes (see, *e.g.*, [26, 27]).

### 2.3 The LFV Decays $l_i \to l_j + \gamma$

As was indicated in the Introduction, in the class of theories we consider, one of the effects of RG running from $M_X$ to $M_R < M_X$ is the generation of new contributions in the amplitudes of the LFV processes [20, 22]. In the "mass insertion" and leading-log approximations (see, *e.g.*, [22, 24, 28]), the branching ratio of $l_i \to l_j + \gamma$ decay due to the new contributions has the following form

$$\mathrm{B}(l_i \to l_j + \gamma) \cong \frac{\Gamma(l_i \to e\nu\bar{\nu})}{\Gamma_{\text{total}}(l_i)}\, \frac{\alpha_{\text{em}}^3}{G_F^2 m_S^8} \left|\frac{(3+a_0^2)m_0^2}{8\pi^2}\right|^2 \left|\sum_k \left(\mathbf{Y}_\nu^\dagger\right)_{ik}\, \ln\frac{M_X}{M_k}\, \left(\mathbf{Y}_\nu\right)_{kj}\right|^2 \tan^2\beta\, , \tag{15}$$

where $i \neq j = 1, 2, 3$, $l_1, l_2, l_3 \equiv e, \mu, \tau$, $m_0$ and $A_0 = a_0 m_0$ are the universal scalar masses and trilinear scalar couplings at $M_X$ and $m_S$ represents SUSY particle mass. It was shown in [28] that in most of the relevant soft SUSY breaking parameter space, the expression

$$m_S^8 \simeq 0.5\; m_0^2\; m_{1/2}^2\; (m_0^2 + 0.6\; m_{1/2}^2)^2\, , \tag{16}$$

$m_{1/2}$ being the universal gaugino mass at $M_X$, gives an excellent approximation to the results obtained in a full renormalisation group analysis, *i.e.*, without using the leading-log and the mass insertion approximations. It proves useful to consider also the "double" ratios,

$$\mathrm{R}(21/31) \equiv \frac{\mathrm{B}(\mu \to e+\gamma)}{\mathrm{B}(\tau \to e+\gamma)}\, \mathrm{B}(\tau \to e\nu_\tau\bar{\nu}_e)\, ,\; \mathrm{R}(21/32) \equiv \frac{\mathrm{B}(\mu \to e+\gamma)}{\mathrm{B}(\tau \to \mu+\gamma)}\, \mathrm{B}(\tau \to e\nu_\tau\bar{\nu}_e)\, , \tag{17}$$

[5] Equation (12) represents the so-called "orthogonal" parametrisation of $\mathbf{Y}_\nu$. In certain cases it is more convenient to use the "bi-unitary" parametrisation [27] $\mathbf{Y}_\nu = \mathbf{U}_R^\dagger \mathbf{Y}_\nu^{\text{diag}}\, \mathbf{U}_L$, where $\mathbf{U}_{\mathrm{L,R}}$ are unitary matrices and $\mathbf{Y}_\nu^{\text{diag}}$ is a real diagonal matrix. The orthogonal parametrisation is better adapted for our analysis and we will employ it in what follows.

which are essentially independent of the SUSY parameters.

To get an estimate for the typical predictions of the schemes with heavy Majorana neutrinos with hierarchical spectrum we will consider further, we introduce a "benchmark SUSY scenario" defined by the values of the SUSY parameters

$$m_0 = m_{1/2} = 250 \text{ GeV}, \quad A_0 = a_0 m_0 = -100 \text{ GeV} , \tag{18}$$

and $\tan\beta \sim (5-10)$. In this scenario the lightest supersymmetric particle is a neutralino with a mass of $\sim$ 100 GeV. The next to the lightest SUSY particles are the chargino and a second neutralino with masses $\sim$ 200 GeV. The squarks have masses in the range of $\sim (400-600)$ GeV. Supersymmetric particles possessing the indicated masses can be observed in the experiments under preparation at the LHC.

The "benchmark" values of $m_0$, $m_{1/2}$ and $A_0$ in eq. (18) correspond to

$$\mathrm{B}(l_i \to l_j + \gamma) \simeq 9.1 \times 10^{-10} \left| \left( \mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu \right)_{ij} \right|^2 \tan^2\beta , \tag{19}$$

where $(L)_{kl} = \delta_{kl}(L)_k$, $L_k \equiv \ln(M_X/M_k)$. Since $\tan^2\beta$ will typically enhance $\mathrm{B}(l_i \to l_j + \gamma)$ by at least 1 order of magnitude, the quantity $\left| \left( \mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu \right)_{21} \right|$ has to be relatively small to be in agreement with the existing experimental upper limit on $\mathrm{B}(\mu \to e + \gamma)$. For given values of the heavy Majorana neutrino masses, this will lead to certain constraints on the parameters in $\mathbf{R}$. Regarding the masses of the heavy Majorana neutrinos, we shall assume that $M_1 \ll M_2 \ll M_3$, with $M_3$ having a value $M_3 \gtrsim (10^{13} - 10^{14})$ GeV, $M_3 \ll M_X$. Constraints from thermal leptogenesis require that $M_1 \gtrsim 10^9$ GeV [33, 34]. This would indicate a hierarchy, *e.g.*, of the form $M_1 \simeq (10^9 - 10^{11})$ GeV, $M_2 \simeq (10^{12} - 10^{13})$ GeV and $M_3 \simeq (10^{14} - 10^{15})$ GeV.

### 2.4 Leptogenesis

In the case of $M_1 \ll M_2 \ll M_3$, the baryon asymmetry of interest is given by

$$Y_B \simeq -10^{-2} \, \kappa \, \epsilon_1 , \tag{20}$$

where $\epsilon_1$ is the CP-violating asymmetry in the decay of the lightest RH Majorana neutrino $N_1$ having the mass $M_1$, and $\kappa$ is an efficiency factor calculated by solving the Boltzmann equations (see, *e.g.*, [33]). A simple approximate expression for the efficiency factor $\kappa$ in the case of thermal leptogenesis we will assume in what follows, was found in [34]:

$$\frac{1}{\kappa} \simeq \frac{3.3 \times 10^{-3}\text{eV}}{\widetilde{m}_1} + \left( \frac{\widetilde{m}_1}{0.55 \times 10^{-3}\text{eV}} \right)^{1.16} , \tag{21}$$

where the neutrino mass parameter $\widetilde{m}_1$ is given by

$$\widetilde{m}_1 \equiv \frac{v_u^2}{M_1} \left( \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \right)_{11} . \tag{22}$$

The CP-violating decay asymmetry $\epsilon_1$ has the form

$$\epsilon_1 \simeq -\frac{3}{8\,\pi} \frac{1}{\left( \mathbf{Y}_\nu \, \mathbf{Y}_\nu^\dagger \right)_{11}} \mathrm{Im} \left\{ \left( \mathbf{Y}_\nu \, \mathbf{Y}_\nu^\dagger \right)_{21}^2 \right\} \frac{M_1}{M_2} . \tag{23}$$

Extensive numerical studies have shown [33, 34] that in MSSM and for hierarchical spectrum of masses of the heavy Majorana neutrinos under discussion, successful thermal leptogenesis is possible only for

$$\widetilde{m}_1 \lesssim 0.12 \text{ eV} . \tag{24}$$

For typical values of $\kappa \sim (10^{-3} - 10^{-1})$ one gets for $Y_B$ a value compatible with the observations [9],

$$Y_B = (6.15 \pm 0.25) \times 10^{-10} , \tag{25}$$

if $\epsilon_1 \sim -(10^{-5} - 10^{-7})$.

As it follows from eqs. (13) and (15), the branching ratios of $l_i \to l_j + \gamma$ decays in the case of interest depend on the orthogonal matrix $\mathbf{R}$. Successful leptogenesis can take place only if $\mathbf{R}$ is complex, so we will consider $(\mathbf{R})^* \neq \mathbf{R}$. In what follows we will use a parametrisation of $\mathbf{R}$ with complex angles (see, *e.g.*, [23, 24]):

$$\mathbf{R} = \mathbf{R}_{12}\,\mathbf{R}_{13}\,\mathbf{R}_{23} \ , \quad \text{or} \quad \mathbf{R} = \mathbf{R}_{12}\,\mathbf{R}_{23}\,\mathbf{R'}_{12} \ , \tag{26}$$

where $\mathbf{R}_{ij}$ ($\mathbf{R'}_{12}$) describes now the rotation with a complex angle $\omega_{ij} = \rho_{ij} + i\sigma_{ij}$ ($\omega'_{12} = \rho'_{12} + i\sigma'_{12}$), $\rho_{ij}$ and $\sigma_{ij}$ ($\rho'_{12}$ and $\sigma'_{12}$) being real parameters. These parametrisations prove particularly convenient for investigating the case of hierarchical spectrum of masses of the heavy RH neutrinos.

# 3 The See-Saw Mechanism, Neutrino Yukawa Couplings, LFV Decays $l_i \to l_j + \gamma$ and Leptogenesis

There has been a considerable theoretical effort in recent years to understand possible connections between the neutrino mass and mixing data, LFV charged lepton decays and leptogenesis. Here we shall focus on the combined constraints which the existing stringent upper limit on the $\mu \to e + \gamma$ decay rate and the requirement of successful leptogenesis impose on MSSMRN. We will be interested, in particular, in the possible implications of these constraints for the form of the matrix $\mathbf{R}$, the heavy Majorana neutrino masses, the predicted rates of the decays $\mu \to e + \gamma$, $\tau \to e + \gamma$ and $\tau \to \mu + \gamma$, and the basic SUSY parameters.

## 3.1 Normal Hierarchical Light Neutrino Mass Spectrum

We set first $m_1$, $M_1$ and $M_2$ to zero. In this approximation we find using $\mathbf{R} = \mathbf{R}_{12}\mathbf{R}_{13}\mathbf{R}_{23}$:

$$\begin{aligned}\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21} \simeq\ & \frac{L_3\, M_3\, \cos\omega_{13}\, \cos\omega_{13}^*}{\sqrt{2}\, v_u^2} \\ & \times \Big[\sqrt{m_2}\, \left(e^{-i\,(\alpha-\beta_M)/2}\, \sqrt{m_3}\, \cos\omega_{23}^* - \sqrt{m_2}\, c_{12}\, \sin\omega_{23}^*\right)\, s_{12}\, \sin\omega_{23} \\ & -m_3\, \cos\omega_{23}\, \cos\omega_{23}^*\, s_{13}\, e^{i\delta}\Big] \ .\end{aligned} \tag{27}$$

In deriving eq. (27) we have set for simplicity $\theta_{23} = \pi/4$ and neglected the terms $\propto m_2 s_{13}$ and $\propto m_{2,3} s_{13}^2$ in the square brackets. The corresponding expressions for $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{31,32}$ are very similar to that for $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}$. In particular, both are proportional to $L_3\, M_3\, \cos\omega_{13}\, \cos\omega_{13}^*$. For the plausible values of $M_3 \cong (10^{14} - 10^{15})$ GeV and $M_X \cong 2 \times 10^{16}$ GeV, one finds that $M_3\, L_3\, \sqrt{\Delta m_{\rm A}^2}/(\sqrt{2} v_u^2) \gtrsim 0.66$. Barring accidental cancellations between the terms in the square brackets in eq. (27), we get from eq. (19) that the predicted value of $\mathrm{B}(\mu \to e+\gamma)$ will be larger at least by a factor of $\sim 10^3$ than the existing upper bound $\mathrm{B}(\mu \to e+\gamma)$. For $M_3 \cong 10^{13}$ GeV and $M_X = 2\times 10^{16}$ GeV one finds $M_3\, L_3 \sqrt{\Delta m^2_{\rm A}/(\sqrt{2} v_u^2)} \cong 0.09$, which for $\tan^2\beta = 25$ (100) and the chosen "benchmark" values of the SUSY parameters leads to $\mathrm{B}(\mu \to e+\gamma) \cong 1.8\times 10^{-10}$ ($7.4\times 10^{-10}$). The values obtained are still larger than the current limit. This might suggest that the SUSY parameter $m_{1/2}$ has a bigger value than the "benchmark scenario" value we have assumed, or that $m_0 \sim m_{1/2} \gtrsim 500$ GeV [6]. We will pursue, however, an alternative hypothesis. We will suppose that $m_{1/2} \sim$ few $\times$ 100 GeV. If indeed $M_3 \gtrsim 5\times 10^{13}$ GeV, $M_3 \ll M_{\rm X}$, where $M_{\rm X} \geq M_{\rm GUT}$, the existing stringent experimental upper limit on $\mathrm{B}(\mu \to e+\gamma)$ might suggest that $\omega_{13} \cong \pi/2$ and we will explore this interesting possibility in what follows. For $\omega_{13} = \pi/2$, the $\mathbf{R}$ matrix has the form

$$\mathbf{R} \simeq \begin{pmatrix} 0 & \sin\omega & \cos\omega \\ 0 & \cos\omega & -\sin\omega \\ -1 & 0 & 0 \end{pmatrix}~, \tag{28}$$

where $\omega \equiv \omega_{12} - \omega_{23}$. Thus, only the combination $(\omega_{12} - \omega_{23})$ of the complex angles $\omega_{12}$ and $\omega_{23}$ appears in the expression for $\mathbf{R}$. It is not difficult to convince oneself that if $m_1$ is negligible, and $\mathbf{R}$ has the form given in eq. (28), we have $(\mathbf{Y}_\nu)_{3j} = 0$ ($j = 1,2,3$). This means that the heaviest RH Majorana neutrino $N_3$ decouples and $\mathbf{Y}_\nu$ coincides in form with the matrix of neutrino Yukawa couplings in the so-called "3×2" see-saw model [48]. We will keep, however, the elements $(\mathbf{Y}_\nu)_{1j(2j)} \neq 0$ ($j = 1,2,3$) in our further analysis.

With $m_1 = 0$ and $\mathbf{R}$ having the form (28), the terms $\sim M_2 L_2$ give the dominant contribution in $\left|\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{ij}\right|$ $(i \neq j)$. We get:

$$\begin{aligned}\left|\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}\right| \;\simeq\; & \frac{L_2\, M_2}{v_u^2}\left|\left(e^{-i\alpha/2}\sqrt{m_2}\, c_{13}\, s_{12}\, c_\omega - e^{-i(\beta_M - 2\delta)/2}\sqrt{m_3}\, s_{13}\, s_\omega\right)\right. \\ & \left.\times \left(e^{i\beta_M/2}\sqrt{m_3}\, c_{13}\, s_{23}\, s_\omega^* - e^{i\alpha/2}\sqrt{m_2}\, c_{12}\, c_{23}\, c_\omega^*\right)\right|~, \end{aligned} \tag{29}$$

where $c_\omega \equiv \cos\omega$, $s_\omega \equiv \sin\omega$ and we have neglected terms $\propto s_{13}$ which can give a correction not exceeding approximately 13%. Setting, for instance $M_2 = 10^{12}$ GeV, we find $|(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu)_{21}| \sim M_2\, L_2 \sqrt{\Delta m^2_{\rm A}/v_u^2} \simeq 10^{-2}$, and correspondingly $\mathrm{B}(\mu \to e+\gamma) \cong 2.3\times 10^{-12}$ for $\tan^2\beta = 25$. This is the range that will be explored by the experiment MEG [19] currently under preparation. Similarly, we obtain for $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{31,32}$:

$$\begin{aligned}\left|\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{31}\right| \;\simeq\; & \frac{L_2\, M_2}{v_u^2}\left|\left(e^{-i\alpha/2}\sqrt{m_2}\, s_{12}\, c_\omega - e^{-i(\beta_M - 2\delta)/2}\sqrt{m_3}\, s_{13}\, s_\omega\right)\right. \\ & \left.\times \left(e^{i\beta_M/2}\sqrt{m_3}\, c_{23}\, s_\omega^* + e^{i\alpha/2}\sqrt{m_2}\, c_{12}\, s_{23}\, c_\omega^*\right)\right|~, \end{aligned} \tag{30}$$

and

$$\begin{aligned}\left|\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{32}\right| \;\simeq\; & \frac{L_2\, M_2}{v_u^2}\left|\left(e^{-i\alpha/2}\sqrt{m_2}\, c_{12}\, c_{23}\, c_\omega + e^{-i\beta_M/2}\sqrt{m_3}\, s_{23}\, s_\omega\right)\right. \\ & \left.\times \left(e^{i\beta_M/2}\sqrt{m_3}\, c_{23}\, s_\omega^* + e^{i\alpha/2}\sqrt{m_2}\, c_{12}\, s_{23}\, c_\omega^*\right)\right|~. \end{aligned} \tag{31}$$

[6] Information about the SUSY parameters $m_0$ and $m_{1/2}$ of interest is expected to be obtained in the experiments under preparation at the LHC.

As it follows from eqs. (29)–(31), for $\omega \neq 0$, the $l_i \to l_j + \gamma$ decay branching ratios of interest depend on the Majorana phase difference $(\alpha - \beta_M)$. The effective Majorana mass $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$-decay (see, *e.g.*, [14, 37]) depends on the same Majorana phase difference (see, *e.g.*, [38, 39]):

$$|\langle m \rangle| \cong \left| \sqrt{\Delta m^2_{21}} \sin^2\theta_{12} e^{i(\alpha - \beta_M)} + \sqrt{\Delta m^2_{31}} \sin^2\theta_{13} \right| . \tag{32}$$

If $s_{13}|s_\omega|$ is not negligibly small, $\mathrm{B}(\mu \to e + \gamma)$ and $\mathrm{B}(\tau \to e + \gamma)$ will depend also on the Dirac phase $\delta$.

For the "double" ratio R(21/31) we find from eqs. (29) and (30):

$$\mathrm{R}(21/31) \cong \frac{\left| \sqrt{m_3}\, s_{23}\, s_\omega - \sqrt{m_2}\, c_{12}\, c_{23}\, c_\omega\, e^{i\frac{\alpha - \beta_M}{2}} \right|^2}{\left| \sqrt{m_3}\, c_{23}\, s_\omega + \sqrt{m_2}\, c_{12}\, s_{23}\, c_\omega e^{i\frac{\alpha - \beta_M}{2}} \right|^2} . \tag{33}$$

Given $\theta_{12}$, $\theta_{23}$ and $m_2/m_3 \cong \sqrt{\Delta m^2_{21}}/\sqrt{\Delta m^2_{31}}$, R(21/31) depends only on $(\alpha - \beta_M)$ and $\omega$. If the terms $\propto \sqrt{m_3}$ ($\propto \sqrt{m_2}$) in eq. (33) dominate, we have $\mathrm{R}(21/31) \cong 1$.

The expression for the double ratio R(21/32) can be obtained from eqs. (29) and (31). For $\sqrt{m_3} s_{13} |s_\omega| \gg \sqrt{m_2} |c_\omega|$ we get $\mathrm{R}(21/32) \cong s^2_{13}/c^2_{23} \lesssim 0.1$, while if $\sqrt{m_3}|s_\omega| \ll \sqrt{m_2}|c_\omega| s_{12}$, one finds $\mathrm{R}(21/32) \cong (\tan^2\theta_{12})/s^2_{23} \cong 0.9$.

### 3.1.1 Leptogenesis Constraints

We shall analyse next the constraints on the parameter $\omega$ in the matrix $\mathbf{R}$, eq. (28), which follow from the requirement of successful thermal leptogenesis. With $\omega_{13} = \pi/2$ we find that in the case of NH light neutrino mass spectrum we are considering.

$$\epsilon_1 \simeq -\frac{3}{8\pi} \left( \frac{m_3 M_1}{v_u^2} \right) \frac{\mathrm{Im}\left[ c^2_\omega + \frac{\Delta m^2_{21}}{\Delta m^2_{31}}\ s^2_\omega \right]}{|c_\omega|^2 + \sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}}\ |s_\omega|^2} \tag{34}$$

$$\simeq \frac{3}{8\pi} \left( \frac{m_3 M_1}{v_u^2} \right) \frac{\sin 2\rho\, \sinh 2\sigma}{(1 - \frac{m_2}{m_3}) \cos 2\rho + (1 + \frac{m_2}{m_3}) \cosh 2\sigma} , \tag{35}$$

where $\rho$ and $\sigma$ are determined by $\omega = \rho + i\sigma$ and we have used $m^2_2 \cong \Delta m^2_{21}$, $m^2_3 \cong \Delta m^2_{31}$ and the relation $\mathrm{Im} c^2_\omega = -\mathrm{Im} s^2_\omega$. Thus, in the case under discussion, the mass $M_2$ governs the magnitude of the $l_i \to l_j + \gamma$ decay branching ratios, whereas $M_1$ determines the value of the leptogenesis decay asymmetry. The conditions $|c_\omega|^2 \geq 0$, $|s_\omega|^2 \geq 0$ imply that $\rho$ and $\sigma$ (in eq. (35)) should satisfy $\cosh 2\sigma \geq |\cos 2\rho|$, which is always valid since $\cosh 2\sigma \geq 1$. As can be easily shown, we have

$$\frac{\left| \mathrm{Im}\left[ c^2_\omega + \frac{\Delta m^2_{21}}{\Delta m^2_{31}}\ s^2_\omega \right] \right|}{|c_\omega|^2 + \sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}}\ |s_\omega|^2} \leq \frac{\left| \mathrm{Im} c^2_\omega \right|}{|c_\omega|^2} \leq 1 , \tag{36}$$

leading to the well-known [49] upper limit

$$|\epsilon_1| \lesssim \frac{3}{8\pi} \left( \frac{m_3 M_1}{v_u^2} \right) \simeq 1.97 \times 10^{-7} \left( \frac{m_3}{0.05\ \mathrm{eV}} \right) \left( \frac{M_1}{10^9\ \mathrm{GeV}} \right) \left( \frac{174\ \mathrm{GeV}}{v_u} \right)^2 . \tag{37}$$

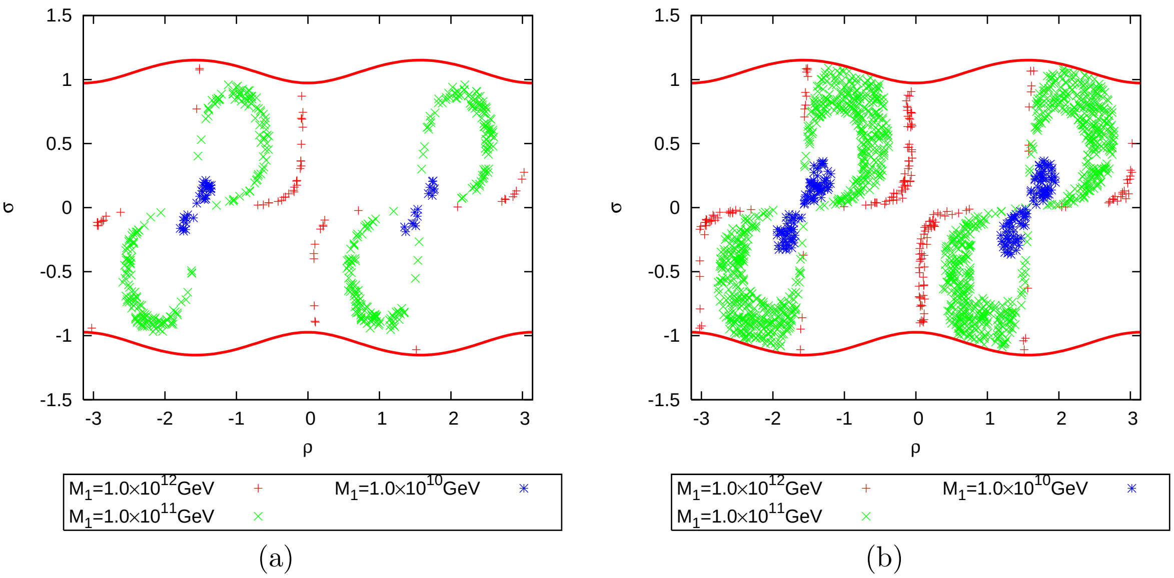


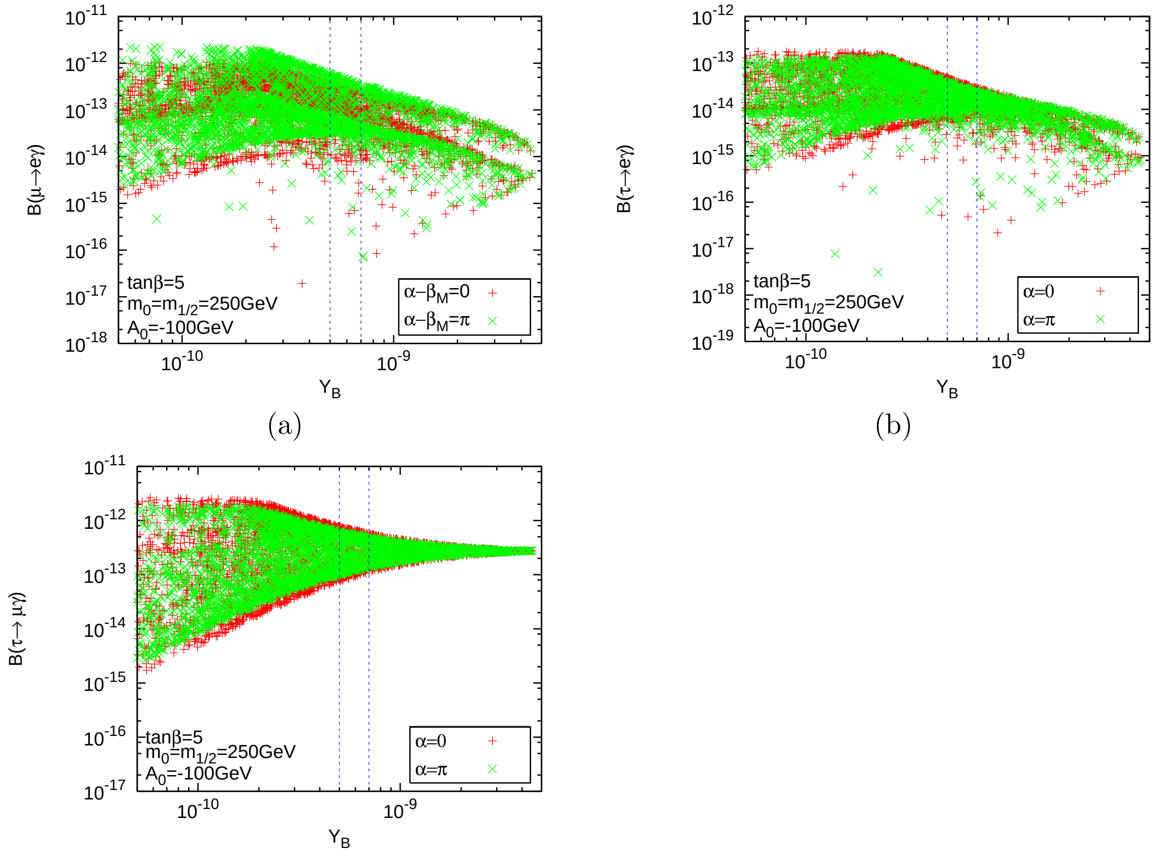

Figure 1: The leptogenesis constraints on the parameters $\rho$ and $\sigma$ for $M_1 = 10^{10}$ GeV; $10^{11}$ GeV; $10^{12}$ GeV (blue; green; red areas) in the case of NH light neutrino mass spectrum. The two panels correspond to two different intervals of values of the baryon asymmetry of the Universe, $Y_B$, considered: (a) $5.0 \times 10^{-10} \le Y_B \le 7.0 \times 10^{-10}$, and (b) $3.0 \times 10^{-10} \le Y_B \le 9.0 \times 10^{-10}$. The solid lines show the limit associated with the upper bound $\widetilde{m}_1 \le 0.12$ eV: outside the region between the solid lines the wash-out effects are too strong and leptogenesis cannot produce the observed baryon asymmetry.

The requirement of a nonzero asymmetry, $\epsilon_1 \neq 0$, implies, as it follows from eq. (35), that both the real and imaginary parts of $\omega$ have to be nonzero, $\rho \neq k\pi/2$, $k = 0, 1, 2, ...$, $\sigma \neq 0$, *i.e.*, that $\mathbf{R}$ has to be complex. Moreover, we should have $\sin 2\rho \sinh 2\sigma < 0$ since the decay asymmetry $\epsilon_1$ has to be negative in order to generate a baryon asymmetry of the correct sign. The maximal asymmetry $|\epsilon_1|$ is obtained for $|\mathrm{Im} c_\omega^2| = |c_\omega|^2$, which is satisfied for $\cos 2\rho\ \cosh 2\sigma = -1$, $\cos 2\rho \neq -1$, $\cosh 2\sigma \neq 1$.

The neutrino mass parameter $\widetilde{m}_1$, eq. (22), can also be easily found:

$$\widetilde{m}_1 \simeq m_3 |c_\omega|^2 + m_2 |s_\omega|^2 = \frac{1}{2}(m_3 + m_2)\cosh 2\sigma + \frac{1}{2}(m_3 - m_2)\cos 2\rho \ge m_2\ . \tag{38}$$

The minimal value of $\widetilde{m}_1 = m_2 \cong \sqrt{\Delta m_{21}^2} \cong 9 \times 10^{-3}$ eV, corresponds to $\cosh 2\sigma = 1$ and $\cos 2\rho = -1$, for which $|\epsilon_1| = 0$. For $9 \times 10^{-3}$ eV $< \widetilde{m}_1 \lesssim 0.12$ eV, where we have taken into account eq. (24), the efficiency factor lies in the interval $1.9 \times 10^{-3} \lesssim \kappa < 3.9 \times 10^{-2}$. For this range of values of $\kappa$ successful leptogenesis is possible for $M_1 \gtrsim 10^{10}$ GeV. We will consider values of $M_1$ in the interval $M_1 = (10^{10} - 10^{12})$ GeV, which is compatible with the assumption we made about the hierarchical mass spectrum of the heavy Majorana neutrinos. Thus, for given $M_1$, the requirement of successful leptogenesis implies a constraint on the two parameters $\rho$ and $\sigma$ of the theory. In Fig. 1 we show the leptogenesis constraint on $\rho$ and $\sigma$ for $M_1 = 10^{10}$ GeV; $10^{11}$ GeV; $10^{12}$ GeV. As we see from Fig. 1, the requirement of successful leptogenesis severely limits the allowed ranges of values of $\rho$ and $\sigma$. Moreover, the values of the two parameters are strongly correlated. We note, in particular, that as $|\sigma|$ increases, the wash-out effects become stronger and for $|\sigma| \gtrsim 1$, the observed baryon asymmetry cannot be reproduced. The maximal asymmetry $|\epsilon_1|$ for a given $M_1$ is obtained for values of $\cos 2\rho$ and $\cosh 2\sigma$ close, but not equal, to $(-1)$ and $(+1)$,




(a) (b) (c)

Figure 2: The correlation between the predicted values of the LFV decay branching ratios B$(\mu \rightarrow e+\gamma)$ (a), B$(\tau \rightarrow e+\gamma)$ (b), B$(\tau \rightarrow \mu+\gamma)$ (c) and of the baryon asymmetry $Y_B$ in the thermal leptogenesis scenario, for $M_1 = 6\times 10^{10}$ GeV, $M_2 = 10^{12}$ GeV and NH light neutrino mass spectrum. The figure was obtained for the "benchmark" values of the soft SUSY breaking parameters $m_0 = m_{1/2} = 250$ GeV, $a_0 m_0 = -100$ GeV and the minimal value of $\tan\beta = 5$. The region between the two vertical dashed lines corresponds to the observed baryon asymmetry: $5.0 \times 10^{-10} \leq Y_B \leq 7.0 \times 10^{-10}$. Results for two values of the Majorana phase $(\alpha - \beta_M)$ equal to 0 (red+green areas) and $\pi$ (green areas) are shown.

respectively. Given the interval of allowed values of $\widetilde{m}_1$, we can write $\widetilde{m}_1 = f\, m_3$ with $f \cong [0.2, 2]$. The condition of maximal $|\epsilon_1|$ implies $\cos 2\rho = (f - \sqrt{1+f^2})m_3/(m_3 - m_2)$. Choosing $f = 1$, *i.e.*, $\widetilde{m}_1 = m_3 \cong 0.05$ eV, for instance, we get $\cos 2\rho \cong -0.5$ and correspondingly $\cosh 2\sigma \cong 2$. In this example $\kappa \cong 5.4 \times 10^{-3}$ and for $\tan^2\beta \gtrsim 3$ we get the requisite value of the the baryon asymmetry for $M_1 \cong 6 \times 10^{10}$ GeV.

In Fig. 2 we show the relation between the predicted values of $Y_B$ in the thermal leptogenesis scenario and of B$(l_i \rightarrow l_j + \gamma)$. The figure was obtained for the "benchmark" values of the soft SUSY breaking parameters $m_0 = m_{1/2} = 250$ GeV, $a_0 m_0 = -100$ GeV and the minimal value of $\tan\beta = 5$. For the relevant heavy Majorana neutrino masses we used $M_1 = 6\times 10^{10}$ GeV and $M_2 = 10^{12}$ GeV. Results for two values of the Majorana phase difference $(\alpha - \beta_M) = 0;\ \pi$, are shown. For the values of $\rho$ and $\sigma$ ensuring successful leptogenesis we find that typically $10^{-14} \lesssim$ B$(\mu \rightarrow e+\gamma) \lesssim 5\times 10^{-13}$, $10^{-13} \lesssim$ B$(\tau \rightarrow \mu+\gamma) \lesssim 5 \times 10^{-12}$ and $10^{-15} \lesssim$ B$(\tau \rightarrow e+\gamma) \lesssim 5 \times 10^{-13}$. However, we have

$B(l_i \to l_j + \gamma) \propto \tan^2\beta$ and, *e.g.*, for $\tan\beta = 20$ we get typically $1.6 \times 10^{-13} \lesssim B(\mu \to e + \gamma) \lesssim 8 \times 10^{-12}$, which is entirely in the range of sensitivity of the MEG experiment. As Fig. 2 indicates, the dependence of $B(l_i \to l_j + \gamma)$ on the Majorana phase $(\alpha - \beta_M)$ is relatively weak.

### 3.2 Inverted Hierarchical Light Neutrino Mass Spectrum

We will perform next a similar analysis assuming that the light neutrino mass spectrum is of the inverted hierarchical type. We set $m_2 \cong m_1$, $m_3/m_{1,2} \cong 0$ and neglect first $M_1/M_3$ and $M_2/M_3$. Setting for simplicity $s_{13} = 0$ and $\theta_{23} = \pi/4$, we find that $B(\mu \to e + \gamma)$ will depend on

$$\begin{aligned}\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21} \;\simeq\; & \frac{-L_3\, M_3\, m_2}{\sqrt{2}} \left(c_{12}\, \sin\omega_{13} + s_{12}\, e^{-i\alpha/2}\, \cos\omega_{13}\, \sin\omega_{23}\right) \\ & \times \left(e^{i\alpha/2}\, c_{12}\, \cos\omega_{13}^*\, \sin\omega_{23}^* - s_{12}\, \sin\omega_{13}^*\right) \,. \end{aligned} \tag{39}$$

This serves to underline that – as in the case of a NH light neutrino spectrum discussed in Section 3.1 – we have typically $\left|\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}\right| \sim M_3 \sqrt{\Delta m^2_{\rm A}}/v_u^2$. This leads for $M_3 \simeq (10^{14} - 10^{15})$ GeV and $m_0$, $A_0$ and $m_{1/2}$ in the few$\times$100 GeV range to a $\mu \to e + \gamma$ decay branching ratio which exceeds the existing limit by approximately 3 orders of magnitude. Looking again for simplifications with interesting phenomenological consequences, we can reduce the magnitude of $\left|\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}\right|$ by setting $\omega_{13} = \omega_{23} = 0$ and, correspondingly, $\mathbf{R}_{13} = \mathbf{R}_{23} = \mathbf{1}$ in eq. (26) [7]. The corresponding form of $R$ is

$$\mathbf{R} \simeq \begin{pmatrix} \cos\omega_{12} & \sin\omega_{12} & 0 \\ -\sin\omega_{12} & \cos\omega_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \,. \tag{40}$$

With $\mathbf{R}$ given by eq. (40) and negligible $m_3/m_{1,2}$, the heaviest (RH) Majorana neutrino $N_3$ decouples and we have again $(\mathbf{Y}_\nu)_{3j} = 0$ $(j = 1, 2, 3)$. Neglecting further the splitting between $m_1$ and $m_2$ we find:

$$\begin{aligned}\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21} \;\simeq\; & -\frac{L_2\, M_2\, \sqrt{|\Delta m^2_{31}|}}{v_u^2} \left(c_{12}\, \sin\omega_{12} - e^{-i\alpha/2}\, s_{12}\, \cos\omega_{12}\right) \\ & \times \left[e^{i\alpha/2}\, c_{12}\, c_{23}\, \cos\omega_{12}^* + (s_{12}\, c_{23} + e^{i\delta}\, c_{12}\, s_{23}\, s_{13})\, \sin\omega_{12}^*\right] \,, \end{aligned} \tag{41}$$

where we have used $m_{1,2} \cong \sqrt{|\Delta m^2_{31}|}$ and have neglected terms $\propto s_{13}$ which give a correction not bigger than approximately 13%. Being of the order $M_2\sqrt{|\Delta m^2_{31}|}/v_u^2$, the expression (41) for $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}$ will lead for $M_2 \simeq 10^{12}$ GeV and values of the soft SUSY breaking parameters in the few$\times$100 GeV range to $B(\mu \to e + \gamma)$ close to the existing limits. For $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{31,32}$ we similarly

[7] If one uses a somewhat different parametrisation of $\mathbf{R}$, namely, $\mathbf{R} = \mathbf{R}_{12}\mathbf{R}_{23}\mathbf{R}'_{12}$, the same result is achieved by setting just $\omega_{23} = 0$.

get

$$\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{31} \simeq -\frac{L_2\, M_2\, \sqrt{\left|\Delta m^2_{31}\right|}}{v_u^2} \left(e^{-i\alpha/2}\, s_{12}\, \cos\omega_{12} - c_{12}\, \sin\omega_{12}\right)$$
$$\times \left[e^{i\alpha/2}\, c_{12}\, s_{23}\, \cos\omega^*_{12} + (s_{12}\, s_{23} - e^{i\delta}\, c_{12}\, c_{23}\, s_{13})\, \sin\omega^*_{12}\right] , \quad (42)$$

$$\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{32} \simeq -\frac{L_2 M_2 \sqrt{\left|\Delta m^2_{31}\right|}}{v_u^2} \left[\left(c_{23}\, s_{12} + e^{-i\,\delta}\, c_{12}\, s_{23}\, s_{13}\right) \sin\omega_{12} + e^{-i\alpha/2}\, c_{12}\, c_{23}\, \cos\omega_{12}\right]$$
$$\times \left[e^{i\alpha/2}\, c_{12}\, s_{23}\, \cos\omega^*_{12} + \left(s_{12}\, s_{23} - e^{i\,\delta}\, c_{12}\, c_{23}\, s_{13}\right) \sin\omega^*_{12}\right] . \quad (43)$$

We see that, as in the case of NH light neutrino mass spectrum, $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}$ and $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{31}$ are rather similar in structure, whereas $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{32}$ differs somewhat. The Majorana phase $\beta_M$ does not appear in the expressions (41)–(43) because we have set $m_3/m_{1,2} = 0$. As the phase factor including the Dirac phase $\delta$ appears always multiplied by the small parameter $s_{13}$, for $s_{13} < 0.1$ the branching ratios depend essentially only on the Majorana phase $\alpha$, which enters also into the expression for the effective Majorana mass $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$-decay [50, 38, 39]:

$$|\langle m \rangle| \cong \sqrt{\Delta m^2_{13}}\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right| . \quad (44)$$

We will give next the ratios of $\mathrm{B}(\mu \to e + \gamma)$ and $\mathrm{B}(\tau \to e + \gamma)$ ($\mathrm{B}(\tau \to \mu + \gamma)$) in the case of negligible contribution of the terms $\propto s_{13}$ [8]:

$$\mathrm{R}(21/31) \simeq \cot^2\theta_{23} , \quad (45)$$
$$\mathrm{R}(21/32) \simeq s_{23}^{-2}\, \frac{\left|e^{i\alpha/2}\, c_{12}\, \sin\omega_{12} - s_{12}\, \cos\omega_{12}\right|^2 \left|c_{12}\, \sin\omega_{12} - e^{i\alpha/2}\, s_{12}\, \cos\omega_{12}\right|^2}{\left|e^{i\alpha/2}\, s_{12}\, \sin\omega_{12} + c_{12}\, \cos\omega_{12}\right|^2 \left|s_{12}\, \sin\omega_{12} + e^{i\alpha/2}\, c_{12}\, \cos\omega_{12}\right|^2} . \quad (46)$$

Hence, as in the case of NH light neutrino mass spectrum, R(21/31) is rather close to one, whereas R(21/32) can have a wide range of values. Most interestingly, R(21/32) can have a value close to two or even be as large as $\sim 10$.

### 3.2.1 Leptogenesis Constraints

We can again work out possible constraints from the requirement of successful leptogenesis. Using expression (40) for the matrix $\mathbf{R}$ and eq. (23) we find that the CP-violating decay asymmetry $\epsilon_1$ of interest has the form

$$\epsilon_1 \simeq -\frac{3}{8\pi}\left(\frac{m_2\, M_1}{v_u^2}\right) \frac{\Delta m^2_{21}}{\left|\Delta m^2_{31}\right|} \frac{\mathrm{Im}\left[\sin^2\omega_{12}\right]}{\left(1 + \frac{\Delta m^2_{21}}{2\left|\Delta m^2_{31}\right|}\right) \left|\sin\omega_{12}\right|^2 + \left|\cos\omega_{12}\right|^2}$$
$$\simeq -\frac{3}{16\pi}\left(\frac{m_2\, M_1}{v_u^2}\right) \frac{\Delta m^2_{21}}{\left|\Delta m^2_{31}\right|}\, \sin 2\rho\, \tanh 2\sigma , \quad (47)$$

[8]For $s_{13} < 0.10$ the correction due to the terms in question can be shown to be smaller than approximately 15%.

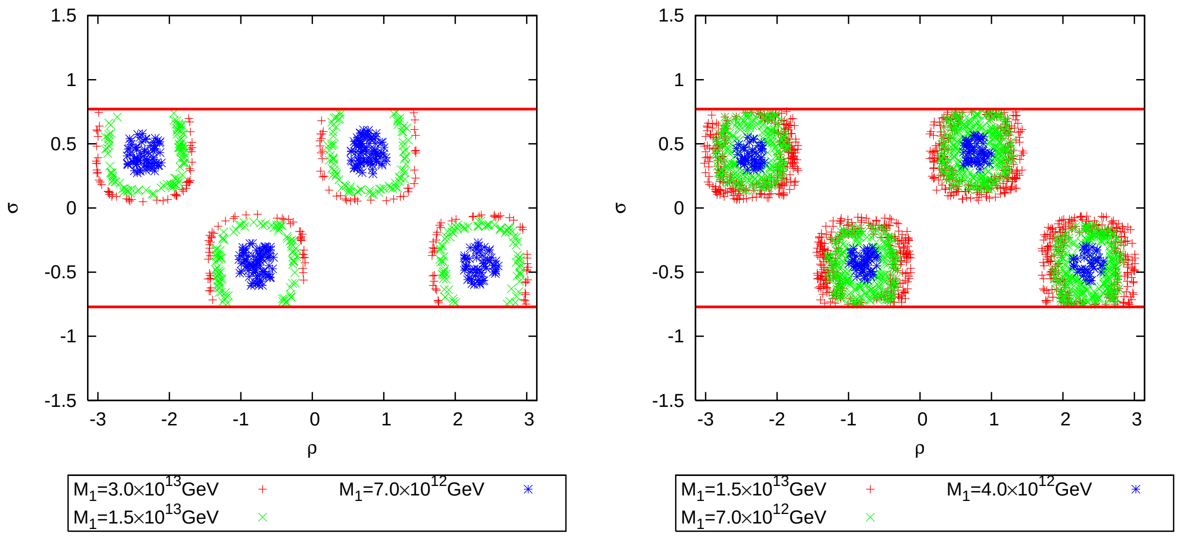



Figure 3: The same as in Fig. 1, but for IH light neutrino mass spectrum and (a) $M_1 = 7 \times 10^{12}$ GeV; $1.5 \times 10^{13}$ GeV; $3 \times 10^{13}$ GeV (blue; green; red areas), (b) $M_1 = 4 \times 10^{12}$ GeV; $7.0 \times 10^{12}$ GeV; $1.5 \times 10^{13}$ GeV (blue; green; red areas).

where $\omega_{12} = \rho + i\sigma$, $m_2 \cong \sqrt{|\Delta m^2_{31}|}$ and we have neglected corrections $\sim \Delta m^2_{21}/\left|\Delta m^2_{31}\right|$. We see that in order to have $\epsilon_1 \neq 0$, both $\rho$ and $\sigma$ should be different from zero: $\rho \neq k\pi/2$, $k = 0, 1, 2...$, $\sigma \neq 0$. Since $\epsilon_1 < 0$, we should have $\sin 2\rho \tanh 2\sigma > 0$.

It follows from eq. (47) that in the case of IH light neutrino mass spectrum under discussion, the CP-asymmetry $\epsilon_1$ is suppressed by the factor $\Delta m^2_{21}/\left|\Delta m^2_{31}\right|$. The expression for the CP-asymmetry we have found for the NH spectrum, eq. (35), does not contain the indicated suppression factor. It is not difficult to show that one always has

$$\frac{\left|\mathrm{Im}\left[\sin^2\omega_{12}\right]\right|}{\left(1 + \dfrac{\Delta m^2_{21}}{2\left|\Delta m^2_{31}\right|}\right)\, |\sin\omega_{12}|^2 + |\cos\omega_{12}|^2} \leq \frac{1}{2}\,. \tag{48}$$

For the asymmetry $\epsilon_1$ we get the upper limit

$$|\epsilon_1| \lesssim \frac{3}{16\pi}\left(\frac{m_2\, M_1}{v_u^2}\right)\frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \simeq 3.2\times 10^{-9}\left(\frac{m_2}{0.05\text{ eV}}\right)\left(\frac{M_1}{10^9\text{ GeV}}\right)\left(\frac{174\text{ GeV}}{v_u}\right)^2, \tag{49}$$

where we have used $\Delta m^2_{21}/\left|\Delta m^2_{31}\right| = 3.2\times 10^{-2}$. The maximal value of $\epsilon_1$ is reached for $\rho = \pi/4$ and $\sigma \gtrsim 0.5$.

For the neutrino mass parameter $\widetilde{m}_1$, eq. (22), we find:

$$\widetilde{m}_1 \simeq m_{1,2}(|\cos\omega_{12}|^2 + |\sin\omega_{12}|^2) = m_{1,2}\cosh 2\sigma \geq m_{1,2}\,. \tag{50}$$

The minimal value of $\widetilde{m}_1 = m_{1,2} \cong \sqrt{|\Delta m^2_{31}|} \cong 5\times 10^{-2}$ eV, corresponds to $\cosh 2\sigma = 1$ for which $|\epsilon_1| = 0$. For $5\times 10^{-2}$ eV $< \widetilde{m}_1 \lesssim 0.1$ eV, the efficiency factor lies in the interval $2.4\times 10^{-3} \lesssim \kappa < 5.4\times 10^{-3}$. It is not difficult to convince oneself that for a given $M_1$, the

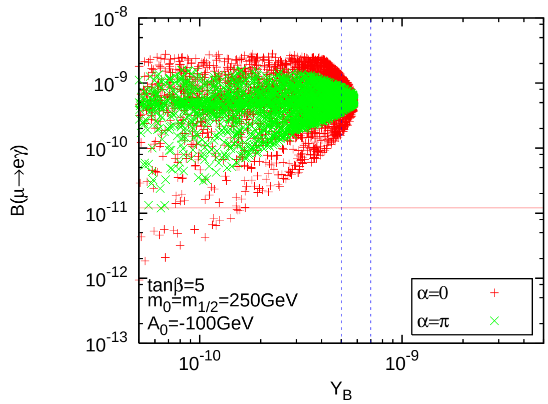
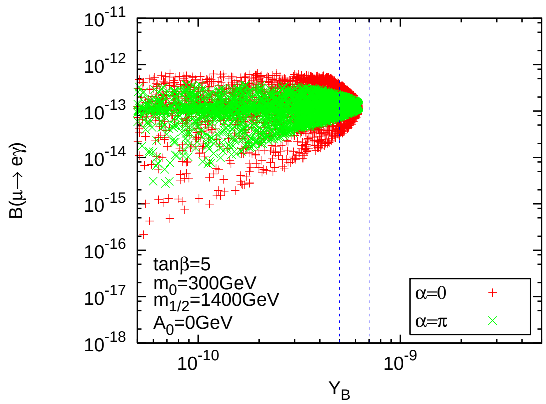
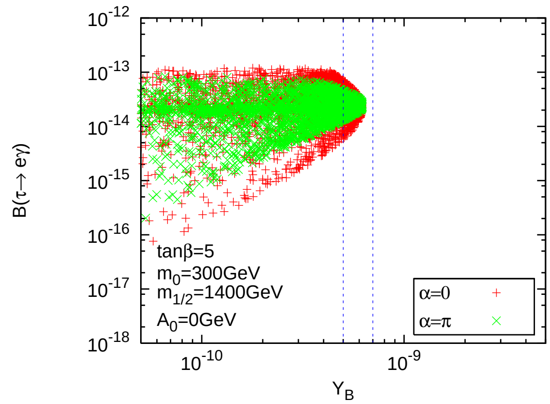



Figure 4: The correlation between the predicted baryon asymmetry of the Universe $Y_B$ and the predicted branching ratio of $\mu \to e + \gamma$ decay $\mathrm{B}(\mu \to e + \gamma)$ in the case of IH light neutrino mass spectrum and for $M_1 = 7.0 \times 10^{12}$ GeV and $M_2 = 4.0 \times 10^{13}$ GeV. The figure was obtained for the "benchmark" values of the soft SUSY breaking parameters $m_0 = m_{1/2} = 250$ GeV, $a_0 m_0 = -100$ GeV and $\tan\beta = 5$. The horizontal line indicates the experimental upper limit on $\mathrm{B}(\mu \to e + \gamma)$, while the region between the two vertical dashed lines is favoured by the observed value of the baryon asymmetry of the Universe, $5.0 \times 10^{-10} \leq Y_B \leq 7.0 \times 10^{-10}$.

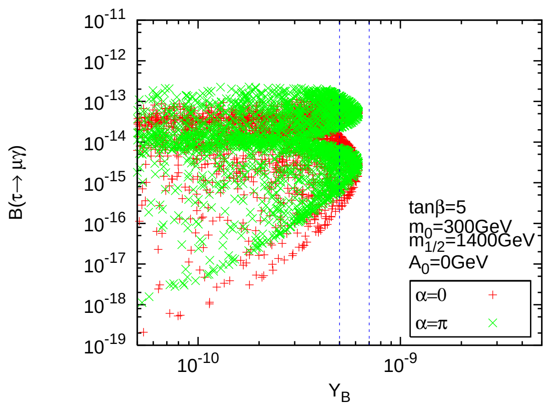
maximal value of $|\epsilon_1|$ is reached for $\sigma \cong \pm 0.5$, for which $\widetilde{m}_1 \cong 7.8 \times 10^{-2}$ eV and, correspondingly, $\kappa \cong 3.2 \times 10^{-3}$. Thus, successful leptogenesis can take place for $M_1 \gtrsim 6.7 \times 10^{12}$ GeV, where we have used eq. (25). In Fig. 3 we show the regions of values of $\rho$ and $\sigma$, favoured by requirement of successful thermal leptogenesis, for three fixed values of $M_1 = 7 \times 10^{12}$ GeV; $1.5 \times 10^{13}$ GeV; $3 \times 10^{13}$ GeV. We find that $|\sigma| \lesssim 0.75$, $|\sigma| \neq 0$. Given the minimal value of $M_1$ determined by the leptogenesis constraint, we will consider further in this subsection the following hypothetical heavy Majorana neutrino mass spectrum: $M_1 = 7.0 \times 10^{12}$ GeV, $M_2 = 4.0 \times 10^{13}$ GeV, $M_3 = 2.0 \times 10^{14}$ GeV. For $M_2 = 4.0 \times 10^{13}$ GeV, we find $M_2 \sqrt{\left|\Delta m^2_{31}\right| L_2 / v_u^2} \cong 0.4$. At the same time the upper limit on $\mathrm{B}(\mu \to e + \gamma)$ implies that for the "benchmark" values of the soft SUSY breaking parameters we have specified earlier and $\tan\beta = 5$, we should have $\left|\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}\right|^2 \lesssim 4.8 \times 10^{-4}$. It follows from the explicit expression for $\left|\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}\right|^2$, eq. (41), that this upper limit is impossible to satisfy for the values of the parameters $\rho$ and $\sigma$ satisfying the leptogenesis constraint (Fig. 3). This is clearly seen in Fig. 4, which shows that the requirement of successful leptogenesis and the existing experimental upper limit on $\mathrm{B}(\mu \to e + \gamma)$ are incompatible in the case of the "benchmark" values of the SUSY parameters, $m_0 = m_{1/2} = 250$ GeV, $a_0 m_0 = -100$ GeV, and of $\tan\beta = 5$. The result we have obtained indicates that in the case of IH light neutrino mass spectrum, the SUSY parameters $m_0$ and/or $m_{1/2}$ should have values considerably larger than the "benchmark" values we consider. More specifically, we can have $m_0 \sim (250 - 300)$ GeV, but $m_0^2 \ll m_{1/2}^2$. This possibility is illustrated in Fig. 5, where the predicted values of $\mathrm{B}(l_i \to l_j + \gamma)$ for $m_0 = 300$ GeV, $m_{1/2} = 1400$ GeV, $a_0 m_0 = 0$, and $\tan\beta = 5$, are shown as functions of the predicted value of the baryon asymmetry. The figure corresponds to $M_1 = 7 \times 10^{12}$ GeV and $M_2 = 4.0 \times 10^{13}$ GeV. Now the requirement for successful leptogenesis is compatible with the existing constraint on $\mathrm{B}(\mu \to e + \gamma)$: for the values of $\rho$ and $\sigma$ ensuring successful leptogenesis we find that typically $3 \times 10^{-14} \lesssim \mathrm{B}(\mu \to e + \gamma) \lesssim 5 \times 10^{-13}$. Significantly larger values of $\mathrm{B}(\mu \to e + \gamma)$ are possible if $\tan\beta \gtrsim 10$. As Fig. 5 also shows, the predicted branching ratios $\mathrm{B}(l_i \to l_j + \gamma)$ exhibit weak




(a)




(b)




(c)

Figure 5: The correlation between the predicted $Y_B$ and the predicted B($\mu \to e+\gamma$) (a), B($\tau \to e+\gamma$) (b), B($\tau \to \mu+\gamma$) (c), for IH light neutrino mass spectrum and $M_1 = 7.0\times10^{13}$ GeV, $M_2 = 4.0\times10^{13}$ GeV. Th results shown are obtained for two values of the Majorana phase $\alpha = 0$; $\pi$ and the following set of values of the SUSY parameters: $m_0 = 300$ GeV, $m_{1/2} = 1400$ GeV, $a_0 = 0$ and $\tan\beta = 5$. The region between the two vertical dashed lines corresponds to $5.0 \times 10^{-10} \leq Y_B \leq 7.0 \times 10^{-10}$ and is favoured by the measured value of the baryon asymmetry. The green (red+green) areas correspond to the Majorana phase $\alpha = \pi$ (0).

dependence on the Majorana phase $\alpha$.

If the light neutrino mass spectrum is of the IH type, the results we have obtained in this subsection can have important implications for the predicted spectrum of SUSY particles in the few 100 GeV − 1 TeV region, to be probed by the experiments at the LHC. For $m_0 = 300$ GeV, $m_{1/2} = 1400$ GeV, $a_0 m_0 = 0$, and $\tan\beta = 5$, the lightest SUSY particle is still a neutralino and its mass is approximately 600 GeV. The mass of next to the lightest SUSY particle, which is a stau, is very close to the mass of the lightest neutralino. At the same time the squarks are predicted to be relatively heavy, having masses $\sim (2-3)$ TeV.

### 3.3 Quasi-Degenerate Light Neutrinos

In this case one has $m_1 \cong m_2 \cong m_3 \equiv m$, with $m \gtrsim 0.1$ eV. It is easy to see that $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}$ will be proportional to $M_3 m/v_u^2$, and therefore a too large branching ratio for $\mu \to e+\gamma$ decay will be predicted. Indeed, setting $M_1 = 0$ and $M_2 = 0$ and using the complex Euler angle parametrisation $\mathbf{R} = \mathbf{R}'_{12}\mathbf{R}_{23}\mathbf{R}_{12}$, we find:

$$\begin{aligned}\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21} &\simeq \frac{L_3 m M_3}{\sqrt{2} v_u^2}\left[-(s_{12} + e^{i\delta} s_{13} c_{12}) \sin\omega_{23}^* \sin\omega_{12}^* - e^{i\frac{\alpha}{2}} c_{12} \cos\omega_{12}^* \sin\omega_{23}^* + e^{i\frac{\beta_M}{2}} c_{13} \cos\omega_{23}^*\right] \\ &\times \left[c_{13}(c_{12} \sin\omega_{12} - e^{-i\frac{\alpha}{2}} s_{12} \cos\omega_{12}) \sin\omega_{23} + e^{-\frac{i}{2}(\beta_M - 2\delta)} s_{13} \cos\omega_{23}\right], \end{aligned} \tag{51}$$

where for simplicity we have set $\theta_{23} = \pi/4$ and have neglected the sub-dominant terms $\propto s_{13}$. There are two possibilities for suppression of $\left|\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}\right|$: $(i)$ If $\sin\omega_{23} = 0$, the contribution due to $M_3$ in $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}$ remains, but is proportional to $s_{13}$. The necessary suppression can take place if $s_{13}$ is sufficiently small. $(ii)$ The parameters $\omega_{12}$ and $\omega_{23}$ can have values such that the different terms $\propto M_3$ in $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}$ cancel (completely or partially) each other. The latter seems to require fine tuning between the values of several very different parameters.

In what follows we shall consider the case $(i)$ and we set $\omega_{23} = 0$. In this case $\mathbf{R}$ has the form given in eq. (40). The quantities of interest $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{ij}$ $(i \neq j)$, including the contributions $\propto M_2$, are given by:

$$\begin{aligned}\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21} &= \frac{L_3 m M_3}{v_u^2} e^{i\delta} s_{23} c_{13} s_{13} \\ &+ \frac{L_2 m M_2}{v_u^2} c_{13}\left[(c_{23} s_{12} + e^{i\delta} s_{23} c_{12} s_{13}) s_\omega^* + e^{i\frac{\alpha}{2}} c_{23} c_{12} c_\omega^*\right](-c_{12} s_\omega + e^{-i\frac{\alpha}{2}} s_{12} c_\omega), \end{aligned} \tag{52}$$

$$\begin{aligned}\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{31} &= \frac{L_3 m M_3}{v_u^2} e^{i\delta} c_{23} c_{13} s_{13} \\ &+ \frac{L_2 m M_2}{v_u^2} c_{13}\left[(-s_{23} s_{12} + e^{i\delta} c_{23} c_{12} s_{13}) s_\omega^* - e^{i\frac{\alpha}{2}} s_{23} c_{12} c_\omega^*\right](-c_{12} s_\omega + e^{-i\frac{\alpha}{2}} s_{12} c_\omega), \end{aligned} \tag{53}$$

$$\begin{aligned}\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{32} &= \frac{L_3 m M_3}{v_u^2} c_{13}^2 c_{23} s_{23} \\ &+ \frac{L_2 m M_2}{v_u^2} c_{13}\left[(-s_{23} s_{12} + e^{i\delta} c_{23} c_{12} s_{13}) s_\omega^* - e^{i\frac{\alpha}{2}} s_{23} c_{12} c_\omega^*\right] \\ &\times \left[(c_{23} s_{12} + e^{-i\delta} s_{23} c_{12} s_{13}) s_\omega + e^{-i\frac{\alpha}{2}} c_{23} c_{12} c_\omega\right], \end{aligned} \tag{54}$$

where $\omega = \omega_{12} + \omega'_{12}$. It is interesting to note that the quantity $\left|\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{32}\right|^2$, and correspondingly $\mathrm{B}(\tau \to \mu + \gamma)$, is not suppressed by the factor $s_{13}^2$. The effective Majorana mass in $(\beta\beta)_{0\nu}$-decay depends in the case of QD spectrum on the CP-violation Majorana phase $\alpha$ [38, 39], present in the expressions (53)–(54) for $\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{ij}$ $(i \neq j)$: $|\langle m \rangle| \cong m \left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|$.

### 3.3.1 Leptogenesis Constraints

For the CP-violating decay asymmetry $\epsilon_1$ we find

$$\epsilon_1 = -\frac{3}{8\pi}\left(\frac{m M_1}{v_u^2}\right)\frac{\Delta m_{21}^2}{m^2}\frac{\mathrm{Im}\left[s_\omega^2\right]}{|c_\omega|^2 + \left(1 + \frac{\Delta m_\odot^2}{2m^2}\right)|s_\omega|^2}. \tag{55}$$

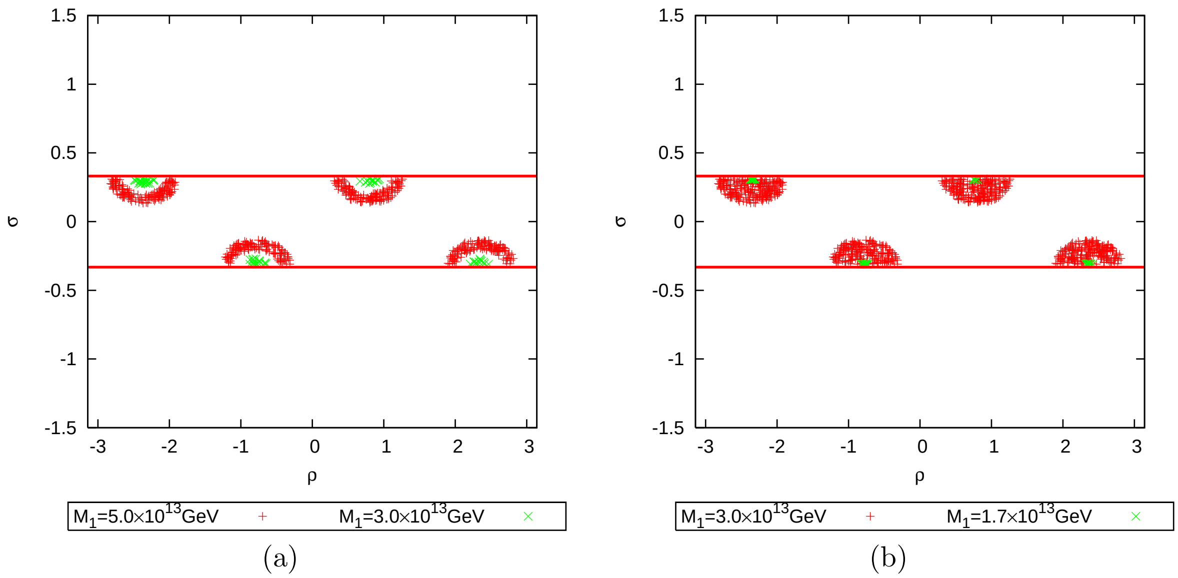



Figure 6: The same as in Fig. 1, but for QD light neutrino mass spectrum and (a) $M_1 = 3.0 \times 10^{13}$ GeV; $5.0 \times 10^{13}$ GeV (green; red areas), (b) $M_1 = 1.7 \times 10^{13}$ GeV; $3.0 \times 10^{13}$ GeV (green; red areas).

It is not difficult to show that

$$\frac{\mathrm{Im}(s_\omega^2)}{|c_\omega|^2 + \left(1 + \frac{\Delta m_\odot^2}{2m^2}\right)|s_\omega|^2} \simeq \frac{\mathrm{Im}(s_\omega^2)}{|c_\omega|^2 + |s_\omega|^2} = \frac{1}{2}\sin 2\rho \tanh 2\sigma \leq \frac{1}{2} \, . \tag{56}$$

Thus, the maximal asymmetry $|\epsilon_1|$ is given by

$$|\epsilon_1| \leq 1.6 \times 10^{-9} \left(\frac{0.1 \text{ eV}}{m}\right)\left(\frac{M_1}{10^9 \text{ GeV}}\right)\left(\frac{174 \text{ GeV}}{v_u}\right)^2 \, . \tag{57}$$

One can easily find also the mass parameter $\widetilde{m}_1$:

$$\widetilde{m}_1 \cong m\left[|c_\omega|^2 + |s_\omega|^2\right] = m\cosh 2\sigma \geq m \, . \tag{58}$$

Since successful leptogenesis is possible for [34] $\widetilde{m}_1 \leq 0.12$ eV, while for QD light neutrino mass spectrum $m \gtrsim 0.1$ eV, we get from eq. (58) that $m \cong 0.1$ eV. Therefore in all further analysis and numerical calculations in this subsection we set $m = 0.1$ eV.

As it follows from the preceding discussion, we have $\widetilde{m}_1 \cong (0.10 - 0.12)$ eV. Correspondingly, the wash-out effect in the case under consideration is relatively strong. Taking into account the precise upper limit on $\widetilde{m}_1$ given in ref. [34], $\widetilde{m}_1 \leq 0.12$ eV, we get for the corresponding efficiency factor $1.9 \times 10^{-3} \lesssim \kappa \lesssim 2.4 \times 10^{-3}$. The condition $\widetilde{m}_1 \leq 0.12$ eV implies $\sigma \lesssim 0.3$, for which $\tanh 2\sigma \lesssim 0.5$. Thus, using eqs. (20) and (57) we obtain the minimal value of $M_1$ ensuring successful leptogenesis: $M_1 \gtrsim 3.0 \times 10^{13}$ GeV. Since $\widetilde{m}_1 \gtrsim 0.1$ eV, we can expect that $\sigma$ lies in the interval $\sigma \cong (0.2 - 0.3)$. This is confirmed by a more detailed numerical analysis. The values of the parameters $\rho$ and $\sigma$ allowed by the leptogenesis constraint are shown in Fig. 6 for $M_1 = 3.0 \times 10^{13}$ GeV; $5.0 \times 10^{13}$ GeV.

Given the lower bound $M_1 \gtrsim 3.0 \times 10^{13}$ GeV, a possible mildly hierarchical heavy Majorana neutrino mass spectrum would correspond to, *e.g.*, $M_1 = 3.0 \times 10^{13}$ GeV, $M_2 = 1.2 \times 10^{14}$ GeV,

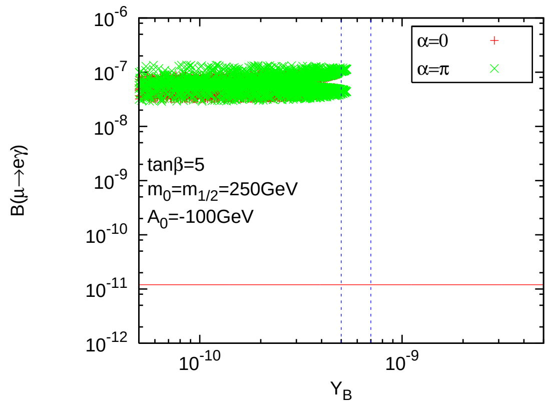
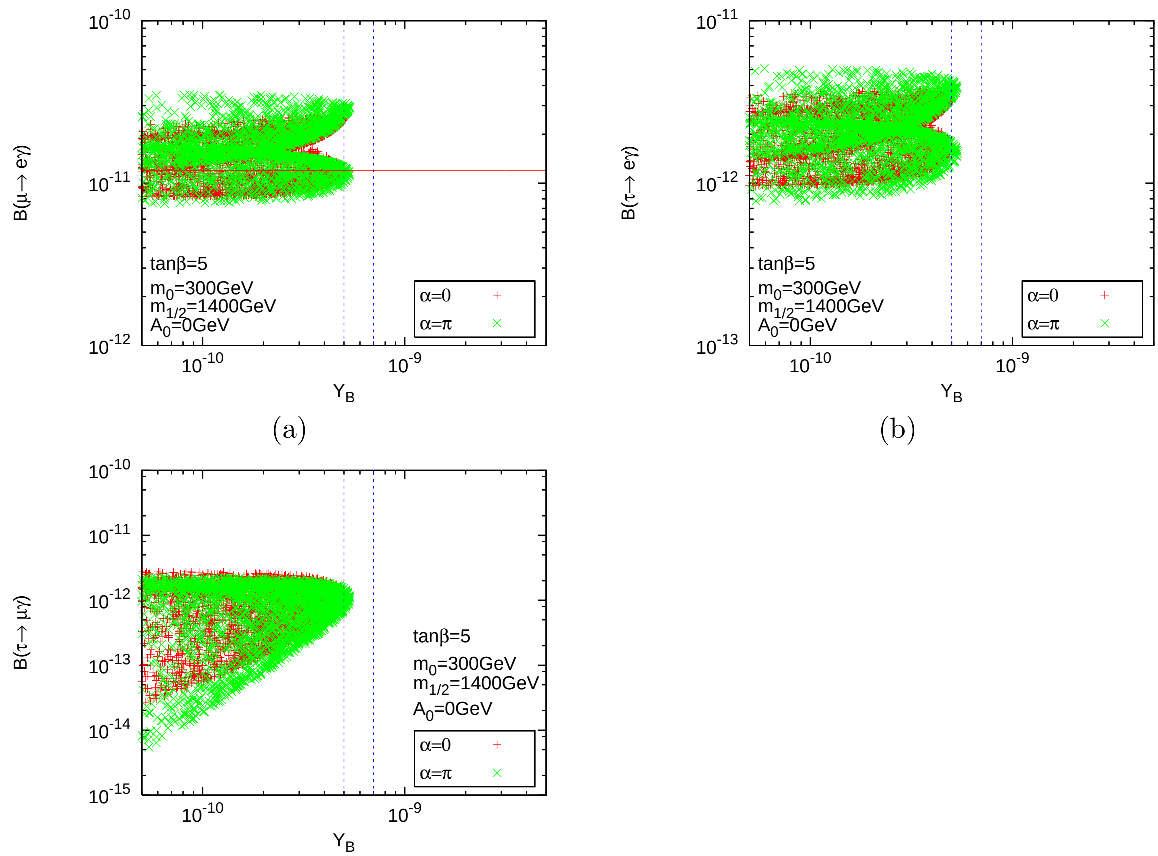



Figure 7: The correlation between the predicted $Y_B$ and the predicted $\mathrm{B}(\mu \to e + \gamma)$ in the case of QD light neutrino mass spectrum and for $M_1 = 3.0 \times 10^{13}$ GeV, $M_2 = 1.2 \times 10^{14}$ GeV and $M_3 = 4.8 \times 10^{14}$ GeV. The "benchmark" values of the soft SUSY breaking parameters $m_0 = m_{1/2} = 250$ GeV, $a_0 m_0 = -100$ GeV and $\tan\beta = 5$, have been used. The horizontal line indicates the experimental upper limit on $\mathrm{B}(\mu \to e + \gamma)$, while the region between the two vertical dashed lines corresponds to $5.0 \times 10^{-10} \leq Y_B \leq 7.0 \times 10^{-10}$ and is favoured by the observed value of $Y_B$.

and $M_3 = 4.8 \times 10^{14}$ GeV. For this spectrum, $L_3 m M_3/v_u^2 \cong 6.0$ and $L_2 m M_2/v_u^2 \cong 2.0$. Using the lowest possible value for $\tan^2\beta \cong 10$ we find that even if $s_{13} = 0$ and the term $\propto M_3$ does not contribute to $\left|\left(\mathbf{Y}_\nu^\dagger L \mathbf{Y}_\nu\right)_{21}\right|$, the contribution of the terms $\propto M_2$ is so large in the case of the "benchmark" values of the soft SUSY breaking parameters, $m_0 = m_{1/2} = 250$ GeV, $a_0 m_0 = -100$ GeV, that the predicted $\mathrm{B}(\mu \to e + \gamma)$ exceeds the existing upper limit on $\mathrm{B}(\mu \to e + \gamma)$ by more than a factor of $\sim 10^3$. The indicated incompatibility between the leptogenesis and $\mathrm{B}(\mu \to e + \gamma)$ constraints is illustrated in Fig. 7.

Similarly to the case of IH light neutrino mass spectrum, the requirement of successful thermal leptogenesis and the upper limit on $\mathrm{B}(\mu \to e + \gamma)$ can be simultaneously satisfied only if the scale of masses of supersymmetric particles is significantly larger than that predicted for the "benchmark" values of the soft SUSY breaking parameters we have adopted. In Fig. 8 we show the correlation between the predicted values of $\mathrm{B}(l_i \to l_j + \gamma)$ for $\sin\theta_{13} = 0.05$ and $\tan\beta = 5$, $m_0 = 300$ GeV, $m_{1/2} = 1400$ GeV and $a_0 = 0$, and the predicted value of the baryon asymmetry. The results presented in this figure have been obtained for the spectrum of the heavy Majorana neutrino masses specified above. We note, in particular, that the predicted interval of values of $\mathrm{B}(\mu \to e + \gamma)$ which is compatible with the observed baryon asymmetry is in the range of sensitivity of the ongoing MEG experiment: $\mathrm{B}(\mu \to e + \gamma)$ can have a value just below the present experimental upper limit. As in the cases of NH and IH light neutrino mass spectra, we find that the dependence of $\mathrm{B}(l_i \to l_j + \gamma)$ on the relevant Majorana phase $\alpha$ is rather weak.

In Fig. 9, the correlations between the predicted value of $Y_B$ and those of the double ratios R(21/31) and R(21/32) are displayed. In the approximation we use, the double ratios are independent of SUSY parameters and are determined only by the off-diagonal elements of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$. When the constraint of successful leptogenesis is imposed, we get for the allowed range of values of R(21/31) for the NH, IH and QD light neutrino mass spectrum respectively $10 \lesssim \mathrm{R}(21/31) \lesssim 100$, $20 \lesssim \mathrm{R}(21/31) \lesssim 50$ and $\mathrm{R}(21/31) \simeq 20;\ 100$. Similarly, for the double ratio R(21/32) we get in the three cases $10^{-2} \lesssim \mathrm{R}(21/32) \lesssim 10$, $10 \lesssim \mathrm{R}(21/32) \lesssim 10^3$ and $50 \lesssim \mathrm{R}(21/32) \lesssim 10^3$, respectively. We find, in particular, that R(21/32) can be much smaller than 1 only for NH light neutrino mass




(a) (b) (c)

Figure 8: The correlation between the predicted $Y_B$ and the predicted B($\mu \to e + \gamma$) (a), B($\tau \to e + \gamma$) (b), B($\tau \to \mu + \gamma$) (c), for QD light neutrino mass spectrum and $M_1 = 3.0 \times 10^{13}$ GeV, $M_2 = 1.2 \times 10^{14}$ GeV, $M_2 = 4.8 \times 10^{14}$ GeV. The SUSY parameters used to obtain the figure are $m_0 = 300$ GeV, $m_{1/2} = 1400$ GeV, $a_0 = 0$ and $\tan\beta = 5$. Results for two values of the Majorana phase $\alpha = 0$; $\pi$ are shown. The region between the two vertical dashed lines corresponds to $5.0 \times 10^{-10} \leq Y_B \leq 7.0 \times 10^{-10}$.

spectrum.

## 4 Conclusions

We have considered the LFV decays $\mu \to e + \gamma$, $\tau \to e + \gamma$ and $\tau \to \mu + \gamma$ and leptogenesis in the MSSM with see-saw mechanism of neutrino mass generation and soft SUSY breaking with universal boundary conditions at a scale $M_{\rm X} > M_{\rm R}$, $M_{\rm R}$ being the heavy (RH) Majorana neutrino mass scale. The heavy Majorana neutrinos were assumed to have hierarchical mass spectrum, $M_1 \ll M_2 \ll M_3$, while the scale $M_{\rm X}$ was taken to be the GUT scale, $M_{\rm X} = 2 \times 10^{16}$ GeV. We have derived the combined constraints, which the existing stringent upper limit on the $\mu \to e + \gamma$ decay rate and the requirement of successful leptogenesis impose on the neutrino Yukawa couplings, heavy Majorana neutrino masses and SUSY parameters in the cases of the three types of light neutrino mass spectrum – normal and inverted hierarchical (NH and IH), and quasi-degenerate (QD). A basic quantity in these analyses is the matrix of neutrino Yukawa couplings, $\mathbf{Y}_\nu$. In the present

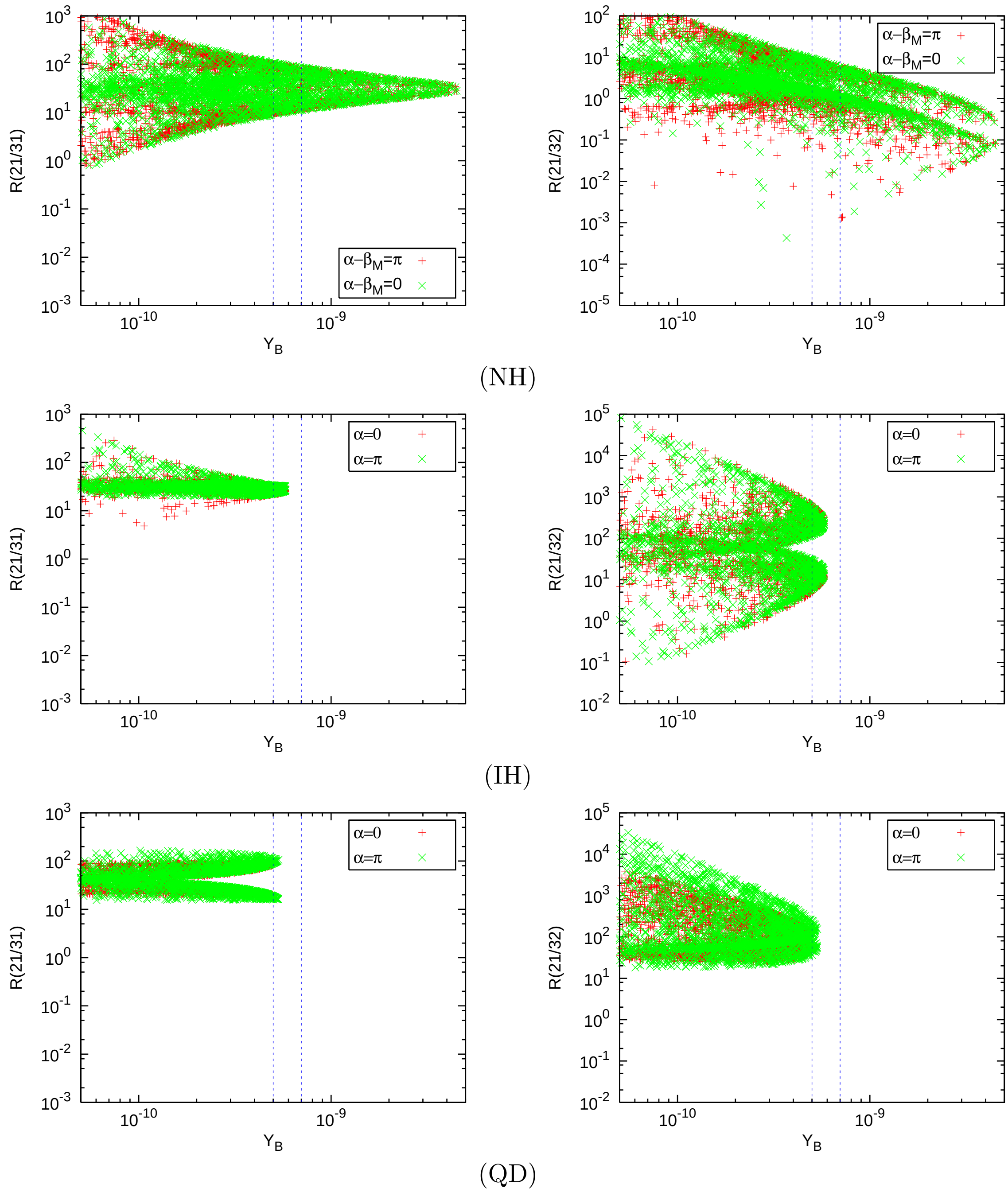



Figure 9: The correlation between the predicted $Y_B$ and the double ratios R(21/31) and R(21/32), defined in eq. (17). The heavy neutrino mass spectrum in the NH, IH, and QD cases is taken to be same as in Fig. 2, Fig. 5 and Fig. 8, respectively. The region between the two vertical dashed lines corresponds to $5.0 \times 10^{-10} \leq Y_B \leq 7.0 \times 10^{-10}$.

work we have used the orthogonal parametrisation of $\mathbf{Y}_\nu$, in which $\mathbf{Y}_\nu$ is expressed in terms of the light neutrino and heavy RH neutrino masses, the PMNS neutrino mixing matrix $U_{\rm PMNS}$, and an orthogonal matrix $\mathbf{R}$. Leptogenesis can take place only if $\mathbf{R}$ is complex.

The constraints from thermal leptogenesis require, in general, that $M_1 \gtrsim 10^9$ GeV. This would indicate a hierarchy, *e.g.*, of the form $M_1 \simeq (10^9 - 10^{11})$ GeV, $M_2 \simeq (10^{12} - 10^{13})$ GeV and $M_3 \gtrsim 10^{13}$ GeV $\gg M_2$, $M_3 < (\ll) M_X$. In our analysis we have considered a "benchmark SUSY scenario" defined by the values of the soft SUSY breaking parameters in the range of few$\times$100 GeV: $m_0 = m_{1/2} = 250$ GeV, $a_0 m_0 = -100$ GeV, and $\tan\beta \sim (5 - 10)$. In this scenario the lightest supersymmetric particle is a neutralino with a mass of $\sim 100$ GeV. The next to the lightest SUSY particles are the chargino and a second neutralino with masses $\sim 200$ GeV. The squarks have masses in the range of $\sim (400 - 600)$ GeV. Using the indicated set of "benchmark" values of the soft SUSY breaking parameters and barring accidental cancellations, we find that for the typical values of the heaviest Majorana neutrino mass $M_3 \cong (5 \times 10^{13} - 10^{15})$ GeV, the limit on the $\mu \to e + \gamma$ decay branching ratio B$(\mu \to e + \gamma)$ is impossible to respect independently of the type of the light neutrino mass spectrum: the predicted B$(\mu \to e + \gamma)$ exceeds the existing upper limit by few orders of magnitude. For each of the three types of neutrino mass spectrum – NH, IH and QD, we find simple forms of the matrix $\mathbf{R}$ which lead to a suppression of the dominant contributions due to the terms $\propto M_3$ in B$(\mu \to e + \gamma)$. In all three cases the matrix $\mathbf{R}$ ensuring the requisite suppression admits a parametrisation by one complex angle. In the case of NH spectrum $\mathbf{R}$ is given by eq. (28), while for IH and QD spectra, $\mathbf{R} \cong \mathbf{R}_{12}$, $\mathbf{R}_{12}$ being the matrix of (complex) rotations in the 1-2 plane. In this case the dominant contribution in B$(\mu \to e + \gamma)$ comes from terms $\propto M_2$. For QD spectrum the terms $\propto M_3$ are suppressed by the factor $\sin\theta_{13}$ and can be comparable to those $\propto M_2$.

The requirement of successful leptogenesis leads to a rather stringent constraint on the complex mixing angle in $\mathbf{R}$. For IH and QD spectra it also implies a relatively large lower limit on the mass of the lightest RH Majorana neutrino: $M_1 \gtrsim 7.0 \times 10^{12}$ GeV and $M_1 \gtrsim 3.0 \times 10^{13}$ GeV, respectively. With such values of $M_1$ and hierarchical heavy Majorana neutrino mass spectrum, the upper bound on B$(\mu \to e + \gamma)$ can be satisfied only if the scale of masses of SUSY particles is considerably higher than that implied by the "benchmark" values of the soft SUSY breaking parameters we have considered. We have analysed a specific case of such SUSY scenario: $m_0 = 300$ GeV, $m_{1/2} = 1400$ GeV and $a_0 = 0$. In this scenario the lightest SUSY particle is a neutralino with a mass of approximately 600 GeV, the next to the lightest SUSY particle is a stau and its mass is very close to the mass of the lightest neutralino, while the squarks are relatively heavy, having masses $\sim (2-3)$ TeV. The predictions for B$(\mu \to e + \gamma)$ are now largely in the range of sensitivity of the ongoing MEG experiment. If more stringent upper limits on B$(\mu \to e + \gamma)$ will be obtained in the future, it would be rather difficult to reconcile the IH and QD light neutrino mass spectra with the $\mu \to e + \gamma$ and leptogenesis constraints and SUSY particle masses in the TeV range. Our results may have important implications for the search of SUSY particles in the few$\times$100 GeV – 1 TeV region, to be performed by the experiments at the LHC.

Satisfying the combined constraints from the existing upper limit on the $\mu \to e + \gamma$ decay rate and the requirement of successful thermal leptogenesis proves to be a powerful tool to test the viability of supersymmetric theories with see-saw mechanism of neutrino mass generation and soft flavour-universal SUSY breaking at a scale above the heavy RH Majorana neutrino mass scale.

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