SISSA 44/2006/EP
TIFR/TH/06-23
TUM-HEP-646/06

Solar Model Parameters and Direct Measurements of Solar Neutrino Fluxes

Abhijit Bandyopadhyay[a)], Sandhya Choubey[b,c)], Srubabati Goswami[b,d)], S. T. Petcov[e)] [1]

[a)] *Tata Institute of Fundamental Research, Mumbai 400005, India*
[b)] *Harish-Chandra Research Institute, Allahabad 211 019, India.*
[c] *Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK.*
[d)] *Physik-Department T30d, Technische Universitaet Muenchen, D-85748 Garching, Germany.*
[e)] *Scuola Internazionale Superiore di Studi Avanzati and*
Instituto Nazionale di Fisica Nucleare, I-34014 Trieste, Italy.

Abstract

We explore a novel possibility of determining the solar model parameters, which serve as input in the calculations of the solar neutrino fluxes, by exploiting the data from direct measurements of the fluxes. More specifically, we use the rather precise value of the ^8B neutrino flux, ϕ_B obtained from the global analysis of the solar neutrino and KamLAND data, to derive constraints on each of the solar model parameters on which ϕ_B depends. We also use more precise values of ^7Be and pp fluxes as can be obtained from future prospective data and discuss whether such measurements can help in reducing the uncertainties of one or more input parameters of the Standard Solar Model.

[1] Also at: Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria.

1 Introduction

There has been a remarkable progress in the studies of solar neutrinos in the last several years. The evidences of solar neutrino ($\nu_{\odot}$) oscillations, obtained first in the Homestake experiment and strengthened by the results of Kamiokande, SAGE and GALLEX/GNO collaborations [1], were made subsequently compelling by the data of Super-Kamiokande (SK), SNO and KamLAND (KL) experiments [2, 3, 4] [2]. The combined charged current (CC) and neutral current (NC) data from SNO and the ν-e^- elastic scattering data from SK experiment showed that the solar ν_e undergo flavour conversion on their way from the central part of the Sun, where they are produced, to the Earth. Under the plausible assumption of CPT-invariance, the results of the KL reactor neutrino experiment [4] established the large mixing angle (LMA) MSW oscillations/transitions as the dominant mechanism at the origin of the observed solar ν_e deficit. The existing global neutrino oscillation data allow us to conclude that the solar ν_e undergo transitions (predominantly) into almost an equal mixture of ν_μ and ν_τ neutrinos. The ratio of the CC and NC event rates observed at SNO provided a measure of the solar ν_e transition probability at energies of $E \sim (5-10)$ MeV, while the SNO NC data permitted to determine with rather good precision the ^{8}B component of the solar ν_e flux [9]. The global solar neutrino data allowed to obtain information on the other important components of the solar neutrino flux - the fluxes of *pp*, *pep* and ^{7}Be neutrinos, and to constrain the flux of CNO neutrinos [10]. The combined solar neutrino and KamLAND data lead to a determination of the neutrino oscillation parameters which drive the solar ν_e oscillations - the neutrino mass squared difference Δm^2_{21} and the mixing angle θ_{12}, with an unexpectedly high precision (see, e.g., [11, 12]).

The latest SNO result on the ^{8}B neutrino flux, ϕ_B, as reported in [9], reads

$$\phi_B^{NC} = 4.94(1 \pm 0.088) \times 10^6 \text{ cm}^{-2}\text{s}^{-1} . \tag{1}$$

This is in good agreement with the Standard Solar Model (SSM) prediction [13]

$$\phi_B^{SSM} = 5.79(1 \pm 0.23) \times 10^6 \text{ cm}^{-2}\text{s}^{-1} . \tag{2}$$

The global oscillation analysis of the solar neutrino and KamLAND data performed in [11], in which ϕ_B is treated as a free parameter, yields the following value of the flux:

$$\phi_B^{Global} = 4.88(1 \pm 0.036) \times 10^6 \text{ cm}^{-2}\text{s}^{-1} . \tag{3}$$

The value of ϕ_B thus obtained corresponds to $f_B = 0.84$, where f_B is defined as

$$f_B = \frac{\phi_B}{\phi_B^{SSM}} . \tag{4}$$

The values of the *pp* and ^{7}Be neutrino fluxes can also be determined similarly from a global analysis of the solar neutrino and KamLAND data, in which the solar luminosity constraint is used [10].

[2]We would like to recall that the hypothesis of neutrino oscillations was formulated in [5]. In [6] it was suggested that the solar ν_e can take part in oscillations involving another active or sterile neutrino. The article [6] appeared before the first data of the Homestake experiment were reported. For a detailed discussion of the evolution of the solar neutrino problem since 1970 and of the variety of different "solutions" proposed over the years, see, e.g., [7, 8].

This analysis showed that with the current data sets the pp neutrino flux can be determined with an uncertainty which is the same as the estimated uncertainty in the SSM prediction for the flux [13]. The uncertainty obtained for the ^7Be flux in the same analysis is larger than the estimated one in the SSM prediction for the same flux [13]. Future high precision measurements of the ^{7}Be neutrino flux by Borexino and KamLAND can lead to a reduction of the uncertainties in both ^7Be and *pp* fluxes determined from the solar neutrino data.

In the present article we explore the possibility of using the precision data (current and prospective) on the i) ^{8}B, ii) ^{8}B and ^{7}Be, and iii) ^{8}B, ^{7}Be and *pp*, solar neutrino fluxes in order to obtain "direct" information (i.e., to constrain or determine) on at least some of the solar model parameters - nuclear reaction rates, opacity, diffusion, heavy element surface abundance, etc., which enter into the calculations of the fluxes in the Standard Solar Model (SSM). It is important to establish whether high precision measurements of the ^{8}B, ^{7}Be and *pp* neutrino fluxes can provide also significant constraints on the indicated SSM parameters because, none of the latter can be determined directly experimentally. The relevant nuclear reaction rates are measured in direct experiments but at energies which are considerably higher than the energies at which the reactions proceed in the central part of the Sun. As a consequence, one has to employ an extrapolation procedure (based on nuclear theory) in order to obtain the values of the rates at the energy of interest. The solar luminosity $L_\odot$ is measured directly with very high accuracy. However, it is still important to determine this fundamental solar observable from solar neutrino flux measurements. The latter provide "real time" information on the rates of the nuclear fusion reactions in the central region of the Sun. Both the photons, observed in the form of solar luminosity, and the neutrinos, emitted by the Sun, are simultaneously produced in these reactions. It takes neutrinos approximately 8 minutes to reach the Earth. In contrast, the "conventionally" measured luminosity of the Sun is determined by photons emitted by the solar surface, which, however, were produced in the central region of the Sun $\sim 4 \times 10^4$ years earlier - the time it takes these photons to reach the surface of the Sun (see, e.g., [14]). Thus, a comparison of the experimentally measured solar luminosity with that obtained from neutrino flux measurements allows, in particular, to test the thermo-nuclear fusion theory of energy generation in the Sun and the hypothesis that the Sun, in what concerns the energy generation, taking place in its interior, and the energy radiation from its surface, is in an approximate steady state.

Another SSM parameter on which the solar neutrino fluxes depend is the ratio of the surface abundance in mass of the elements heavier than helium and of the surface abundance (in mass) of hydrogen (surface heavy element composition), Z/X. At present there is rather large uncertainty in the SSM estimated value of this parameter (see, e.g., [13, 15]), as will be discussed in somewhat greater detail further. Moreover, the estimated uncertainty in the value of Z/X is obtained [13, 15] assuming that the total spread of all modern determinations of Z/X is equal to the 3σ uncertainty in Z/X. Clearly, a determination of the surface element composition parameter Z/X from neutrino flux measurements could be very helpful for resolving the indicated problems. It could also be very useful for solar model building.

In the analysis that follows we will use the SSM by Bahcall and Pinsonneault from 2004 [13] as a "benchmark" solar model for the predictions of the solar neutrino fluxes and the estimated uncertainties in these predictions, originating from the different SSM parameters.

2 Preliminary Observations

There are six principal nuclear reactions and decays in which neutrinos are produced in the Sun (see, e.g., [16]). Four of them generate neutrinos with continuous energy spectrum. These are the fusion of two protons (pp ν's), and the decays of the nuclei ^8B (^8B ν's), ^{13}N and ^{15}O (CNO ν's). The other two, the fusion of two protons and electron and the capture of an electron by a ^7Be nucleus produce neutrino lines (the so-called pep and ^7Be ν's). The shapes of the energy spectra of the pp, ^8B and CNO neutrinos are determined by nuclear physics and are well known. However, the SSM predictions for the total values of the pp, pep, ^7Be, ^8B and the CNO neutrino fluxes depend on several SSM input parameters. The uncertainties associated with these parameters lead to (normalisation) uncertainties in the predicted fluxes.

There are altogether 11 input SSM parameters on which the SSM predictions for the ^8B, ^7Be, pp and the CNO solar neutrino fluxes in general depend [13]. These are first of all the S-factors (see, e.g., [17]) of the nuclear reactions $^1H(p,e^+\nu_e)^2H$, $^3He(^3He,2p)^4He$,$^3He(^4He,\gamma)^7Be$, $^{14}N(p,\gamma)^{15}O$, $^7Be(p,\gamma)^8B$, and of e^- capture on ^7Be. They are standardly denoted respectively by S_{11}, S_{33}, S_{34}, $S_{1,14}$, S_{17} and S_{e^-7}. The additional parameters are directly related to the physics of the Sun: they are [13] the solar luminosity, $L_\odot$, age, $\tau_\odot$, opacity, $O_\odot$, diffusion, $D_\odot$, and the ratio of the mass fractions of the elements heavier than helium and of hydrogen at the surface of the Sun (surface composition), Z/X . The type of dependence of a given solar neutrino flux (^8B, ^7Be, pp, pep and CNO) on a specific SSM parameter varies with the flux. For the three fluxes of interest for our further discussion, for instance, we have [17]:

$$\begin{aligned}\phi_B = C_B\ (S_{11})^{-2.59}(S_{33})^{-0.40}(S_{34})^{+0.81}(S_{1,14})^{+0.01}(S_{17})^{+1.0}(S_{e^-7})^{-1.0} \\ \times(L_\odot)^{+6.76}(\tau_\odot)^{+1.28}(O_\odot)^{-2.93}(D_\odot)^{-2.20}(Z/X)^{+1.36}\ ,\end{aligned} \tag{5}$$

$$\phi_{Be} = C_{Be}\ (S_{11})^{-0.97}(S_{33})^{-0.43}(S_{34})^{+0.86}(L_\odot)^{+3.40}(\tau_\odot)^{+0.69}(O_\odot)^{-1.49}(D_\odot)^{-0.96}(Z/X)^{+0.62}\ , \tag{6}$$

$$\phi_{pp} = C_{pp}\ (S_{11})^{+0.14}(S_{33})^{+0.03}(S_{34})^{-0.06}(S_{1,14})^{-0.02}(L_\odot)^{+0.73}(\tau_\odot)^{-0.07}(O_\odot)^{+0.14}(D_\odot)^{+0.13}(Z/X)^{-0.08}\ , \tag{7}$$

where C_B, C_{Be} and C_{pp} are constants.

The nuclear reaction S-factors of interest are measured typically at c.m. energies exceeding ~ 500 keV, which are at least by a factor ~ 25 larger than those corresponding to the conditions under which the reactions take place in the central region of the Sun. As a consequence, the results on the S-factors obtained experimentally have to be extrapolated to the much lower energy of ~ 20 keV, which is of interest for the solar neutrino flux calculations. The extrapolation procedure brings additional theoretical uncertainty in the S-factor values. This uncertainty can be substantial for a given S-factor.

Because of the importance of the S_{17}-factor for the prediction of ϕ_B and the interpretation of the data of the Homestake, SK and SNO experiments, considerable efforts have been made to determine it with a relatively high precision. This was done using data i) from a direct measurement of the cross-section of the reaction $p +^7 Be \rightarrow^8 B + \gamma$ [18], and ii) of indirect studies of the same reaction (via the Coulomb dissociation process $\gamma +^8 B \rightarrow p +^7 Be$ [19], and heavy-ion transfer and breakup processes [20, 21]). In the SSM calculations of the solar neutrino fluxes [13] the most precise result reported in [18] is used. The value recommended in [18] reads:

$$S_{17}(0) = 21.4 \pm 0.5\ (expt)\ \pm 0.6\ (theo)\ \text{eV b}, \tag{8}$$

the quoted 1σ error being smaller than 5% [3]. However, from a more recent data, obtained with radioactive ion beams, the following value was found in [21]: $S_{17}(0) = 18.2 \pm 1.7$ eV b. One of the theoretical uncertainties in the value of S_{17} quoted above, eq. (8), is associated with the extrapolation method used to obtain this result. In [23] it is argued that a larger extrapolation error, than is usually taken into account, should be assigned in the evaluation of the uncertainties in $S_{17}(0)$.

According to ref. [13], the largest contribution to the uncertainties in the predictions of the fluxes of neutrinos produced in the reactions of the *pp*-chain in the Sun (^8B, ^7Be, pp, pep), is due to the uncertainty in the knowledge of the S-factor S_{34}. The estimated 1σ uncertainty in S_{34} is approximately 9.4% [13]. The S_{34} factor has been measured recently [24] at four values of the c.m. energy from the interval (400 - 950) keV. Using an extrapolation of earlier and the new results to an energy of ~ 20 keV, the authors of [24] obtain $S_{34} = 0.53 \pm 0.02 \pm 0.01$. If this result will be confirmed, that would lead to a reduction of the 1σ uncertainty in the value of S_{34} approximately by a factor of 2. The LUNA collaboration is planning to perform a more precise measurement of S_{34} in an experiment which is under preparation at the Gran-Sasso National Laboratory in Italy [25].

In the case of the CNO neutrinos, the largest nuclear physics uncertainty is associated with the uncertainty in the value of the cross-section of the reaction $p +^{14} N \rightarrow^{15} O + \gamma$, which is directly related to the S-factor $S_{1,14}$ (see Table 2).

In what concerns the uncertainties in the predictions of the solar neutrino fluxes due to the other non-nuclear physics parameters, the largest, according to the SSM BP04 [13], is a consequence of the lack of sufficiently precise knowledge of the surface element composition of the Sun. New values for the abundances in mass of the elements C, N, O, Ne and Ar have been derived [26] using three-dimensional rather than one-dimensional model of the solar atmosphere, including hydrodynamical effects, etc. The new abundance estimates together with the best-estimates for other solar abundances [27] imply $Z/X = 0.0176$, which is considerably smaller than the earlier result [27] $Z/X = 0.0229$. The new values of the solar surface abundances of C, N, O, Ne and Ar, when incorporated into solar models, lead to serious discrepancies with helioseismological data [13, 28, 15] [4]. The estimated uncertainty in the value of Z/X, according to [13], is approximately 15% (see Table 2). The latter is obtained [13, 15] [5] assuming that the total spread of all modern determinations of Z/X is equal to the 3σ uncertainty in Z/X.

It is pertinent to point out that the knowledge of the solar neutrino oscillation parameters Δm^2_{21} and $\sin^2\theta_{12}$ with relatively small uncertainties can be crucial for a successful high precision

[3] The earlier recommended value [22], $S_{17}(0) = 19^{+4}_{-2}$ eV b, had a 1σ uncertainty of approximately 15%.

[4] For this reason the authors of our "benchmark" SSM BP04 did not include the new most recent estimate of the parameter Z/X in the calculations of the solar neutrino fluxes. The possible effects of this new result on the SSM predictions of the solar neutrino fluxes were considered in [28, 15].

[5] The authors of [15] made a comment worth quoting concerning the uncertainties in the element abundances under discussion: "Estimating the uncertainty in an abundance determination is even more difficult than arriving at a best-estimate abundance.".

determination of the fluxes of solar ^8B, ^7Be and pp neutrinos. The existing data allow a determination of Δm^2_{21} and $\sin^2\theta_{12}$ at 3σ with an error of approximately 11% and 25%, respectively. Much higher precision can (and most likely will) be achieved in the future. The data from phase-III of the SNO experiment [9] using ^{3}He proportional counters for the neutral current rate measurement could lead to a reduction of the error in $\sin^2\theta_{12}$ to 21% [29, 30]. If instead of 766.3 t yr one uses simulated 3 kt yr KamLAND data in the same global solar and reactor neutrino data analysis, the 3σ errors in Δm^2_{21} and $\sin^2\theta_{12}$ diminish to 7% and 18% [30]. The most precise measurement of Δm^2_{21}, could be achieved [29] using Super-Kamiokande doped with 0.1% of Gadolinium (SK-Gd) for detection of reactor $\bar{\nu}_e$ [31]: the SK detector gets the same flux of reactor $\bar{\nu}_e$ as KamLAND and after 3 years of data-taking, Δm^2_{21} could be determined with an error of 3.5% at 3σ [29]. A dedicated reactor $\bar{\nu}_e$ experiment with a baseline $L \sim 60$ km, tuned to the minimum of the $\bar{\nu}_e$ survival probability, could provide the most precise determination of $\sin^2\theta_{12}$ [32]: with statistics of ~ 60 GW kt yr and a systematic error of 2% (5%), $\sin^2\theta_{12}$ could be measured with an accuracy of 6% (9%) at 3σ [30]. The inclusion of the uncertainty in θ_{13} ($\sin^2\theta_{13} <$0.05) in the analyzes increases the quoted errors by (1–3)% to approximately 9% (12%) [30]. Even higher precision in the measurement of Δm^2_{21} and $\sin^2\theta_{12}$ can be reached with one module (of 147 kt fiducial mass) of the water Čerenkov detector MEMPHYS doped with 0.1% of Gadolinium (MEMPHYS-Gd), and with a 50 kt scale liquid scintillator detector (LENA), installed in the Frejus underground laboratory [33]. The improved determination of Δm^2_{21} and θ_{12} with KamLAND or dedicated post-KamLAND reactor neutrino experiments has been studied also, e.g., in refs. [34, 35, 36], whereas the potential improvements of the precision on these parameters from future solar neutrino experiments has been investigated, e.g., in refs. [30, 32, 10].

3 Present Knowledge and Future Measurements of Solar Neutrino Fluxes

Precise knowledge of solar neutrino fluxes is a key ingredient of our analysis. In this Section, we summarise the presently existing data and information on solar neutrino fluxes and the improvements in the determination of the fluxes that are possible in the future.

Among all the eight solar neutrino fluxes, at present we only have a direct experimental determination of the total ^8B neutrino flux produced inside the Sun, through the measurement of the rate of the neutral current reaction on deuterium in the SNO experiment (cf. eq. (1)). The 1σ uncertainty in the value of the ^8B neutrino flux determined from the NC SNO data is approximately 8.8% (see eq. (1)). Global oscillation analyses of solar and KamLAND data, in which the ^8B neutrino flux is treated as a free parameter, allow to determine ϕ_B with even higher precision, as eq. (3) shows. In Fig. 1 we present the range of allowed values of f_B (defined in eq. (4)), obtained in analyzes of the current data and of prospective data from future experiments. The corresponding 1σ uncertainties in f_B are given in Table 1. Combining the results from other solar neutrino experiments with the NC data from SNO improves the precision of determination of the solar neutrino oscillations parameters Δm^2_{21} and $\sin^2\theta_{12}$ and hence reduces the 1σ error on the ^8B neutrino flux to 4.4%. This is further reduced to 3.6% by addition of the KamLAND results in the global data analysis. Also shown in Fig. 1 and Table 1 are the expected uncertainty on

Data set used	1σ uncertainty in f_B (in %)
Solar	4.4
Solar + KamLAND	3.6
Solar + SNO-III + KamLAND	3.2
Solar + SNO-III + pp + KamLAND	2.5
Solar + SNO-III + pp + SPMIN	1.7

Table 1: 1 σ uncertainties of f_B

f_B with inclusion of prospective data from future planned/proposed experiments. The phase-III of SNO is expected to provide direct (uncorrelated) measurement of the NC event rate with a precision greater than that achieved in the earlier salt phase. For the third phase data from SNO we have assumed the same central values of the CC and NC event rates as those observed during the salt phase, but smaller uncertainties in the measured CC and NC rates, namely, 4.0% and 6.4% respectively [37]. With the inclusion of the indicated prospective results from this phase (referred to as SNO-III in Fig. 1 and Table 1), the 1σ uncertainty in f_B could be reduced to 3.2%. This uncertainty would further diminish to 2.5% and 1.7% if we added successively to the analysis the prospective data from a "generic *pp*" and from "SPMIN" experiments. The "generic *pp*" experiment in Fig. 1 and Table 1 refers to a high precision $\nu - e^-$ elastic scattering experiment [6] with assumed 1% error in the measured reaction rate. The latter is simulated at the best-fit values of $\Delta m^2_{21} = 8 \times 10^{-5}$ eV^2 and $\sin^2\theta_{12} = 0.31$. The "SPMIN" refers to a reactor experiment with a baseline of 60 km, tuned to the Survival Probability MINimum [32]. The results given in Fig. 1 and Table 1 for this experiment correspond to statistics of 3 kt yr of data and a systematic error of 2%. We refer the reader to [30, 32] for further details of our method used for the analysis of projected data from future experiments.

The SSM prediction for ^7Be neutrino flux reads:

$$\phi^{SSM}_{Be} \;=\; 4.86(1 \pm 0.12) \times 10^9 \text{ cm}^{-2}\text{sec}^{-1} , \tag{9}$$

the estimated uncertainty being 12%. The flux of low energy *pp* neutrinos is calculated very precisely within the SSM - the estimated uncertainty is 1%:

$$\phi^{SSM}_{pp} \;=\; 5.94(1 \pm 0.01) \;\times 10^{10}\text{cm}^{-2}\text{sec}^{-1} . \tag{10}$$

The ^7Be and *pp* neutrino fluxes can be determined in a solar model independent way together with the ^8B neutrino flux by treating all three fluxes as free parameters in global oscillation analyzes [10]. Below we summarise the results obtained for the ^7Be and *pp* neutrino fluxes using present and future prospective data from solar neutrino experiments [10]. The results for ϕ_{pp} depend on whether the luminosity constraint [39] is included in the analysis or not; the determination of ϕ_{Be} is essentially independent of the luminosity constraint [10]. Without employing the

[6]There are a number of planned sub-MeV solar neutrino experiments (LowNu Experiments) aiming to observe and measure directly the *pp* neutrino flux using either charged current reactions (LENS, MOON, SIREN [38]) or the $\nu - e^-$ elastic scattering process (XMASS, CLEAN, HERON, MUNU, GENIUS [38]).

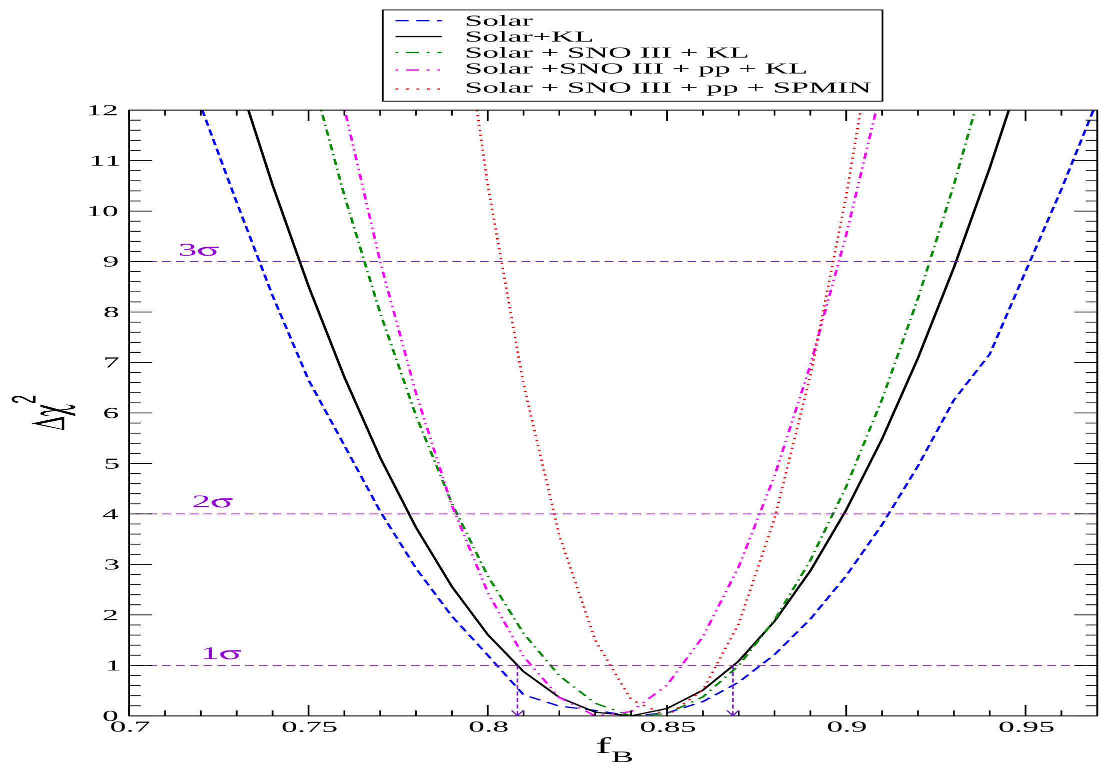

Figure 1: The dependence of $\Delta\chi^2$ on f_B, showing the range of allowed values of f_B, determined using the currently existing data and prospective data from future experiments.

luminosity constraint, both ϕ_{Be} and ϕ_{pp} are determined from the existing data with a precision which is much worse than the estimated precision of the BP04 SSM predictions for the two fluxes. The inclusion of the luminosity constraint in the analysis brings about a drastic improvement in the determination of ϕ_{pp}: even with the present solar and KamLAND data, ϕ_{pp} is determined with an uncertainty of approximately 2% at 1σ.

Improvement in the determination of ϕ_{Be} will be possible from a direct measurement of the ^7Be neutrino flux, as is envisaged in the forthcoming Borexino solar neutrino [40] and KamLAND experiments [41]. Data from the Borexino experiment with a total 1σ error of 10% could lead to a determination of the ^7Be neutrino flux with a 1σ uncertainty of $\sim$ 10%, which is somewhat smaller than the estimated uncertainty in the BP04 SSM prediction for ϕ_{Be} (see eq.(9)). With a total uncertainty of 5% in the measured event rate in Borexino, ϕ_{Be} is expected to be determined with 1σ error of 5.5%, while a measurement of the Borexino event rate with an error of 3% could lead to a factor of three improvement in the precision on ϕ_{Be} with respect to the currently estimated 12% uncertainty in the SSM prediction for ϕ_{Be}.

Precision data from the Borexino experiment is expected to bring also a significant improvement in the precision of determination of ϕ_{pp}. As long as the luminosity constraint is imposed, a measurement of the Borexino event rate with a 5% error could lead to a determination of ϕ_{pp} with a 1σ uncertainty of 0.5%. The addition of a high precision data from a "generic pp" experiment is not expected to lead to any significant reduction of the uncertainty in the value of ϕ_{pp} as long as the luminosity constraint is taken into account [10]. However, the data from these experiments could certainly improve noticeably the precision of the pp flux determination, which can be achieved without using the luminosity constraint. Moreover, it should be possible to test the applicability of the photon luminosity constraint itself and, more generally, the thermo-nuclear theory of the energy generation in the Sun, by comparing the measured value of the solar photon luminosity with the value obtained using the results from the LowNu and the other solar neutrino experiments [10].

4 Determining the SSM Input Parameters from Direct Solar Neutrino Flux Measurements

In the theoretical framework of the SSM, the dependence of the solar neutrino flux from the i^{th} nuclear process, ϕ_i, on the input parameters of the SSM is given, as we have already seen on the examples of the ^8B, ^7Be and pp fluxes, by power laws:

$$\phi_i \;\; = \;\; C_i \times \prod_{\text{all } j} x_j^{\alpha_{ij}} \; . \tag{11}$$

Here C_i is a constant and α_{ij} in the exponent is the logarithmic derivative of ϕ_i with respect to the SSM input parameter x_j,

$$\alpha_{ij} \;\; = \;\; \frac{\partial \ln \phi_i}{\partial \ln x_j}. \tag{12}$$

The values of the different logarithmic derivatives for the BP04 SSM [13] are given in Table 2. We have, in general, eight such equations for the eight different solar neutrino fluxes [7].

Let us first consider a general formulation of the problem where we assume that we have a total of N input parameters in the solar model calculations and that out of the total eight fluxes, K different fluxes along with their 1σ uncertainties can be determined from direct measurements. We can then pick up the set of K equations from the above set (11) and can solve them for any subset $x_{j_1}, x_{j_2}, .., x_{j_K}$ of K different input parameters of the solar model. For the rest of (N-K) parameters x_j we can use the values ${x_j}^0$ found in the SSM. We can express the constants C_i's in terms ϕ_i^{SSM}'s and x_j^0's using the equation

$$\phi_i^{SSM} \;\; = \;\; C_i \times \prod_{\text{all } j} (x_j^0)^{\alpha_{ij}} \; . \tag{13}$$

[7] In this counting we have included also the fluxes of hep and ^{17}F solar neutrinos [13], which, however, are predicted to be exceedingly small. These two fluxes do not play any significant role in the analyses of the presently existing solar neutrino data.

Thus, we write the set of K equations for the fluxes $\phi_{i_1}, \phi_{i_2}, ..., \phi_{i_K}$ as

$$\begin{aligned} \frac{\phi_i}{\phi_i^{SSM}} &= \frac{\prod_{j=j_1,j_2,..,j_K} (x_j)^{\alpha_{ij}} \prod_{j\neq j_1,j_2,..,j_K} (x_j^0)^{\alpha_{ij}}}{\prod_{\text{all } j} (x_j^0)^{\alpha_{ij}}} \\ &= \prod_{j=j_1,j_2,..,j_K} \left(\frac{x_j}{x_j^0}\right)^{\alpha_{ij}}, \qquad i = i_1, i_2, .., i_K \; . \end{aligned} \tag{14}$$

Taking logarithm on both sides of eq. (14) we get

$$\ln\left(\frac{\phi_i}{\phi_i^{SSM}}\right) = \sum_{j=j_1,j_2,..,j_K} \alpha_{ij} \ln\left(\frac{x_j}{x_j^0}\right), \qquad i = i_1, i_2, .., i_K \; . \tag{15}$$

The above set of K equations can be written in a matrix form as

$$\begin{pmatrix} \alpha_{i_1j_1} & \alpha_{i_1j_2} & .. & \alpha_{i_1j_K} \\ \alpha_{i_2j_1} & \alpha_{i_2j_2} & .. & \alpha_{i_2j_K} \\ . & . & . & . \\ . & . & . & . \\ \alpha_{i_Kj_1} & \alpha_{i_Kj_2} & .. & \alpha_{i_Kj_K} \end{pmatrix} \begin{pmatrix} \ln\frac{x_1}{x_1^0} \\ \ln\frac{x_2}{x_2^0} \\ . \\ . \\ \ln\frac{x_K}{x_K^0} \end{pmatrix} = \begin{pmatrix} \ln\frac{\phi_1}{\phi_1^{SSM}} \\ \ln\frac{\phi_2}{\phi_2^{SSM}} \\ . \\ . \\ \ln\frac{\phi_K}{\phi_K^{SSM}} \end{pmatrix} \tag{16}$$

A general solution of eq. (16) is given by

$$\ln\left(\frac{x_r}{x_r^0}\right) = \frac{DetX_r}{DetA}, \qquad r = j_1, j_2, .., j_K \; , \quad \text{Det}A \neq 0 \; , \tag{17}$$

where

$$A = \begin{pmatrix} \alpha_{i_1j_1} & \alpha_{i_1j_2} & .. & \alpha_{i_1j_K} \\ \alpha_{i_2j_1} & \alpha_{i_2j_2} & .. & \alpha_{i_2j_K} \\ . & . & . & . \\ . & . & . & . \\ \alpha_{i_Kj_1} & \alpha_{i_Kj_2} & .. & \alpha_{i_Kj_K} \end{pmatrix} \tag{18}$$

and X_r is the matrix obtained by replacing the r^{th} column of the matrix A by the column matrix in the right hand side of eq. (16). Thus, we can write the solution for the set of K parameters of the solar model $\{x_r\}$, $r = j_1, j_2, .., j_K$, in the form

$$x_r = x_r^0 \exp\left(\frac{DetX_r}{DetA}\right), \qquad r = j_1, j_2, .., j_K \; . \tag{19}$$

To evaluate the uncertainties in each of the K SSM parameters of the set $\{x_j\}$, $j = j_1, j_2, .., j_K$, determined using the data on the solar neutrino fluxes, we take the derivative of the logarithm of both sides of eq. (11) for $i = i_1, i_2, .., i_K$:

$$\delta \ln \phi_i = \sum_{\text{all } j} \alpha_{ij} \delta \ln x_j \; , \qquad i = i_1, i_2, .., i_K \; . \tag{20}$$

This set of K equations can also be written in matrix form as

$$\begin{pmatrix} \alpha_{i_1 j_1} & \alpha_{i_1 j_2} & .. & \alpha_{i_1 j_K} \\ \alpha_{i_2 j_1} & \alpha_{i_2 j_2} & .. & \alpha_{i_2 j_K} \\ . & . & . & . \\ . & . & . & . \\ \alpha_{i_K j_1} & \alpha_{i_K j_2} & .. & \alpha_{i_K j_K} \end{pmatrix} \begin{pmatrix} \delta \ln x_{j_1} \\ \delta \ln x_{j_2} \\ . \\ . \\ \delta \ln x_{j_K} \end{pmatrix} = \begin{pmatrix} \delta \ln \phi_{i_1} - \sum_{j \neq j_1,..,j_K} \alpha_{i_1 j} \delta \ln x_j \\ \delta \ln \phi_{i_2} - \sum_{j \neq j_1,..,j_K} \alpha_{i_2 j} \delta \ln x_j \\ . \\ . \\ \delta \ln \phi_{i_K} - \sum_{j \neq j_1,..,j_K} \alpha_{i_K j} \delta \ln x_j \end{pmatrix} \tag{21}$$

A general solution of eq. (21) is then given by

$$\delta \ln x_r = \frac{\mathrm{Det} D_r}{\mathrm{Det} A} , \qquad r = j_1, j_2, .., j_K , \quad \mathrm{Det} A \neq 0 , \tag{22}$$

where A is the matrix given by eq. (18) and D_r is the matrix obtained by replacing the r^{th} column of the matrix A by the column matrix appearing in the right hand side of eq. (21). Obviously, the right hand side of eq. (22) should be a linear combination of the quantities $\frac{\delta \phi_i}{\phi_i}$, $i = i_1, i_2, .., i_K$, and $\frac{\delta x_j}{x_j}$, $j \neq j_1, j_2, .., j_K$, and therefore eq. (22) can be written as

$$\delta \ln x_r = \sum_{i=i_1,..,i_K} P_{ir} \delta \ln \phi_i + \sum_{j \neq j_1,..,j_K} Q_{jr} \delta \ln x_j , \qquad r = j_1, j_2, .., j_K , \tag{23}$$

where the coefficients P_{ir} and Q_{jr} involve the different logarithmic derivatives α_{ij}. It follows from the preceding equation that the 1σ relative uncertainty in x_r is given by

$$\begin{aligned} \Delta \ln x_r &= \sqrt{\sum_{i=i_1,..,i_K} P_{ir}^2 \left(\Delta \ln \phi_i\right)^2 + \sum_{j \neq j_1,..,j_K} Q_{jr}^2 \left(\Delta \ln x_j^0\right)^2} \\ \text{or,} \quad \ln\left(1 + \frac{\Delta x_r}{x_r}\right) &= \sqrt{\sum_{i=i_1,..,i_K} P_{ir}^2 \left[\ln\left(1 + \frac{\Delta \phi_i}{\phi_i}\right)\right]^2 + \sum_{j \neq j_1,..,j_K} Q_{jr}^2 \left[\ln\left(1 + \frac{\Delta x_j^0}{x_j^0}\right)\right]^2} \\ \text{or} \quad \frac{\Delta x_r}{x_r} &= \left| 1 - \exp\left(\sqrt{\sum_{i=i_1,..,i_K} P_{ir}^2 \left[\ln\left(1 + \frac{\Delta \phi_i}{\phi_i}\right)\right]^2 + \sum_{j \neq j_1,..,j_K} Q_{jr}^2 \left[\ln\left(1 + \frac{\Delta x_j^0}{x_j^0}\right)\right]^2}\right)\right| \end{aligned} \tag{24}$$

where $\Delta \phi_i / \phi_i$ is the 1σ relative uncertainty in the directly measured flux ϕ_i and $\Delta x_j^0 / x_j^0$ is the 1σ relative uncertainties of the parameter x_j^0 as estimated in the SSM.

5 Determining One SSM Input Parameter Using Measured ^8B Neutrino Flux

Following the technique described in Section 4, we can use the measured value of the ^8B neutrino flux, ϕ_B, and its uncertainty, as given in eq. (3), to determine one of the parameters x_{j_1} of the

j	$\alpha_{pp,j}$	$\alpha_{pep,j}$	$\alpha_{hep,j}$	$\alpha_{Be,j}$	$\alpha_{B,j}$	$\alpha_{N,j}$	$\alpha_{O,j}$	$\alpha_{F,j}$
S_{11}	+0.14	−0.17	−0.08	−0.97	−2.59	−2.53	−2.93	−2.94
S_{33}	+0.03	+0.05	−0.45	−0.43	−0.40	+0.02	+0.02	+0.02
S_{34}	−0.06	−0.09	−0.08	+0.86	+0.81	−0.05	−0.05	−0.05
$S_{1,14}$	−0.02	−0.02	−0.01	0	+0.01	+0.85	+1.00	+0.01
S_{17}	0	0	0	0	+1.00	0	0	0
$L_\odot$	+0.73	+0.87	+0.12	+3.40	+6.76	+5.16	+5.94	+6.25
Z/X	−0.08	−0.17	−0.24	+0.62	+1.36	+1.99	+2.06	+2.17
$\tau_\odot$	−0.07	0	−0.11	+0.69	+1.28	+1.01	+1.27	+1.29
$O_\odot$	+0.14	+0.24	+0.54	−1.49	−2.93	−1.81	−2.25	−2.35
$D_\odot$	+0.13	+0.22	+0.38	−0.96	−2.20	−2.86	−3.10	−3.22
S_{e^-7}	0	0	0	0	−1.00	0	0	0

Table 2: Values of the logarithmic derivatives $\alpha_{ij} = \frac{\partial \ln \phi_i}{\partial \ln x_j}$, corresponding to different solar neutrino fluxes ϕ_i (pp, pep, hep, $^7Be, j$, ^8B, ^{13}N, ^{15}O, ^{17}F) and the input parameters of the solar model (from ref. [13]).

solar model together with its uncertainty, $\frac{\Delta x_{j_1}}{x_{j_1}}$. They are given respectively by eqs. (19) and (24) reduced to K = 1:

$$x_{j_1} = x^0_{j_1} \left(\frac{\phi_B}{\phi_B^{SSM}}\right)^{\frac{1}{\alpha_{B,j_1}}} , \tag{25}$$

$$\frac{\Delta x_{j_1}}{x_{j_1}} = \left| 1 - \exp\left(\sqrt{\left(\frac{1}{\alpha_{B,j_1}}\right)^2 \left[\ln\left(1 + \frac{\Delta\phi_B}{\phi_B}\right)\right]^2 + \sum_{j\neq j_1}\left(\frac{\alpha_{B,j}}{\alpha_{B,j_1}}\right)^2 \left[\ln\left(1+\frac{\Delta x^0_j}{x^0_j}\right)\right]^2}\right)\right| \tag{26}$$

We show results for the central value of the input parameters with respect to their values used in BP04 SSM and their corresponding uncertainties in Table 3. The extreme right column of Table 3 gives the factor by which the central values of different parameters will change with respect to their values as used in the BP04 SSM [13] if we use the measured ^8B neutrino flux, given in eq. (3), for their calculations. Note that if the measured mean value of ϕ_B differs from the value predicted by the SSM, ϕ_B^{SSM}, the value of the parameter x_{j_1} obtained using eq. (25) would differ from its SSM predicted value $x^0_{j_1}$ by a factor controlled by the corresponding logarithmic derivative $\alpha_{B,j_1} = \frac{\partial \ln \phi_B}{\partial \ln x_{j_1}}$.

We present in Table 3 the uncertainty of each of the SSM input parameters, evaluated from eq. (26), for three different "benchmark" values of the uncertainty in the measured ^8B neutrino flux. For comparison we have also included in column 5 of Table 3 the estimated uncertainties in the SSM parameters in the BP04 SSM. It follows from eq. (26) that the relative uncertainty in the parameter x_{j_1} depends on the inverse power of the magnitude of the corresponding logarithmic derivative α_{B,j_1}. It should be clear from eq. (5) and Table 2 that since the relevant logarithmic derivatives in the cases of the S-factors S_{33} and $S_{1,14}$ are relatively small, these quantities cannot be determined with smaller uncertainties by using even a high precision measurement of the ^{8}B neutrino flux. We therefore do not show the results for these cases in Table 3. It follows from the results reported in Table 3, in particular, that the SSM parameters like S_{11}, Z/X, $L_\odot$ and $O_\odot$ can be determined with uncertainties less than 10% owing to the relatively large values of their corresponding logarithmic derivatives. For the uncertainty in diffusion parameter $D_\odot$ we get approximately 10.6%. The table shows also that using a direct high precision measurement of the ^8B neutrino flux can allow to determine the parameter Z/X with an uncertainty which is smaller than its currently estimated uncertainty in the BP04 SSM [13]. At the same time, the parameter S_{17} is determined much less precisely - with uncertainty of 25%, in spite of the fact that $\phi_B \propto S_{17}$.

We see from Table 3 that the uncertainties of most of the SSM parameters under discussion are essentially stable when the 1σ error in the measured ^8B neutrino flux changes from 2% to 4%. The reason for such a behavior is that for very small values of $\Delta\phi_B/\phi_B$, the first term in the right hand side of eq. (26) is much smaller than the second term which controls the uncertainty $\Delta x_{j_1}/x_{j_1}$. In Fig. 2 we have plotted the fractional uncertainty in each of the SSM parameters as a function of the 1σ error in the measured value of the ^8B neutrino flux [8]. Figure 2 shows that if the ^8B flux uncertainty is larger than $\sim 5\%$, the first term in eq. (26) would dominate over the second and $\Delta x_j/x_j$ can exhibit a stronger dependence on the uncertainty $\Delta\phi_B/\phi_B$. The degree of this dependence is controlled by the corresponding α_{Bj_1} value. However, as we have shown before (cf. Fig. 1 and Table 1), the uncertainty in the value of f_B, determined from the current data, is already approximately 4%. The second term in eq. (26) is dominant and we do not expect any significant improvement of the precision of determination of the SSM input parameters with more precise measurement of ϕ_B alone. Small improvements can nonetheless be expected, especially in what concerns the precision of determination of Z/X.

6 Determining Two SSM Input Parameters Using Measured ^8B and ^7Be Neutrino Fluxes

As we have discussed in Section 3, a relatively high precision measurement of the ^7Be neutrino flux can be performed by the Borexino experiment. The KamLAND experiments can also provide valuable data on ϕ_{Be}. One could use the experimentally measured values of the ^8B and ^7Be neutrino fluxes to determine any two of the SSM input parameters [9]. Equations (19) and (24) for

[8]Obviously, the uncertainty in the value of a given SSM input parameter determined exploiting a solar neutrino flux measurement depends also on the uncertainties in the remaining SSM input parameters used in the evaluation.

[9]We have also examined the uncertainties of the SSM parameters one obtains if only high precision prospective data on the ^7Be neutrino flux is used to determine the parameters. We have found, in particular, that S_{33} and S_{34} can be determined with a precision of 43.3% (33%) and 17.2% (12%), respectively, if the ^7Be neutrino flux is measured with 1σ error of 10% (2%). This should be compared with the uncertainties of 76% and 30% in S_{33} and S_{34} we have obtained in the previous Section, using ^8B neutrino flux measurement with 4% uncertainty. Even if we take the 1σ error in ϕ_{Be} to be 2%, all the other SSM parameters are detrmined with uncertainties which are larger than those we found in Section 5 when the same parameters are determined from the measured ϕ_B with 1σ error of 4%.

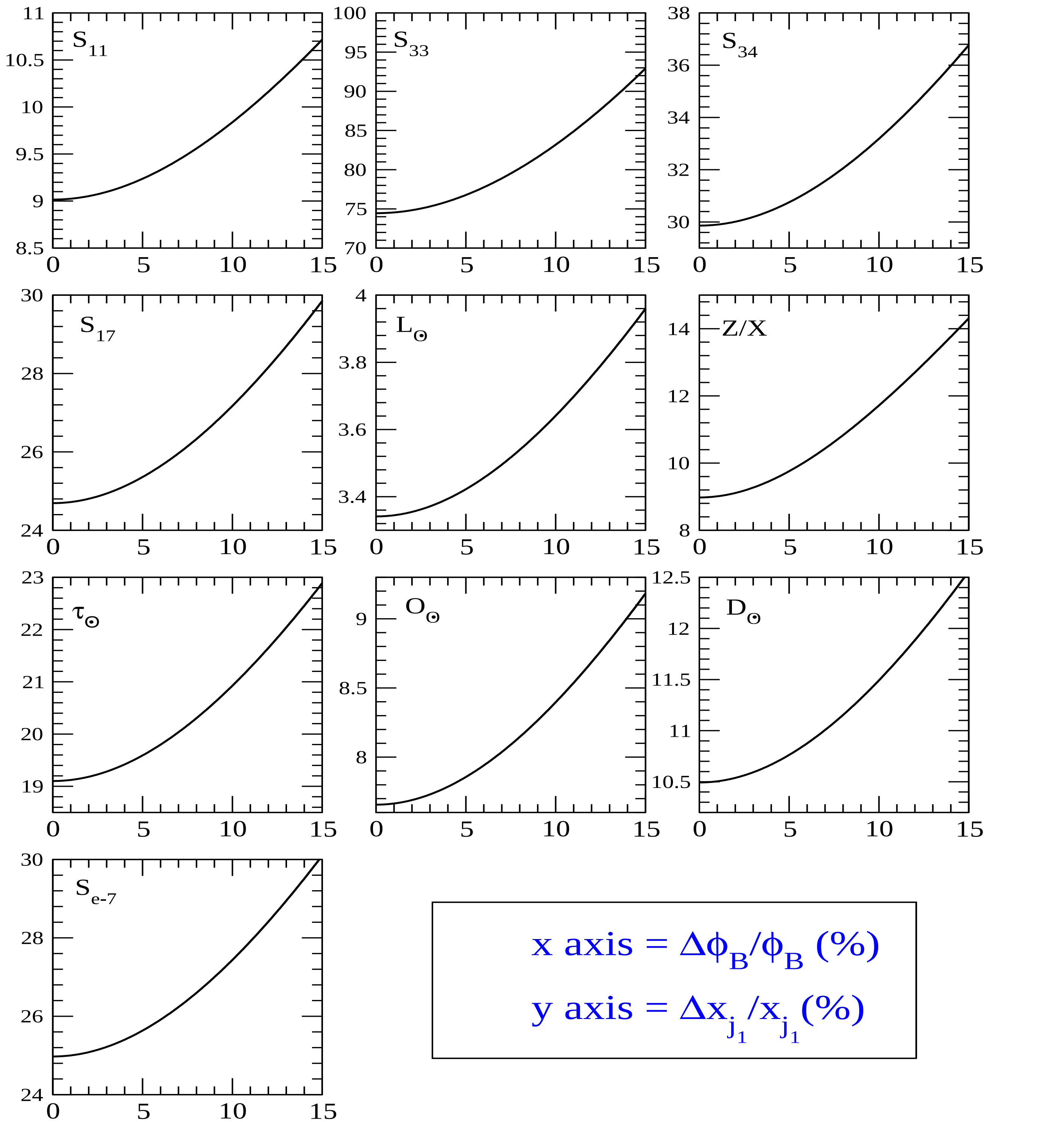

Figure 2: The 1σ fractional uncertainty $(\frac{\Delta x_{j_1}}{x_{j_1}})$ of the various SSM input parameters as a function of the uncertainty in the measured ^8B neutrino flux $(\Delta\phi_B/\phi_B)$.

Name of model parameters (x_{j_1})	$\frac{\Delta x_{j_1}}{x_{j_1}}$ (%)			$\frac{\Delta x^0_{j_1}}{x^0_{j_1}}$ (%)	$\frac{x_{j_1}}{x^0_{j_1}}$
	$\frac{\Delta\phi_B}{\phi_B}=2\%$	$\frac{\Delta\phi_B}{\phi_B}=3\%$	$\frac{\Delta\phi_B}{\phi_B}=4\%$		
S_{11}	9.05	9.10	9.16	0.4	1.07
S_{34}	30.01	30.19	30.44	9.4	0.81
S_{17}	24.80	24.94	25.12	3.8	0.84
Luminosity $L_\odot$	3.35	3.37	3.39	0.4	0.97
Z/X	9.11	9.27	9.48	15.0	0.88
Age of sun	19.18	19.28	19.42	0.4	0.87
Opacity $O_\odot$	7.69	7.73	7.79	2.0	1.06
Diffusion $D_\odot$	10.54	10.59	10.66	2.0	1.08
S_{e^-7}	25.08	25.22	25.40	2.0	1.19

Table 3: The uncertainties for each of the SSM input parameters, which are expected to be obtained if a high precision direct measurement of the ^8B neutrino flux is used to determine the corresponding SSM parameter. Given are also the uncertainties in the SSM input parameters [13], used in the SSM calculations of the solar neutrino fluxes.

K=2 give the central values and the uncertainties for any two parameters x_{j_1} and x_{j_2} as

$$x_{j_1} = x^0_{j_1} \times \left[\left(\frac{\phi_B}{\phi_B^{SSM}}\right)^{\alpha_{Be,j_2}} \times \left(\frac{\phi_{Be}}{\phi_{Be}^{SSM}}\right)^{-\alpha_{B,j_2}}\right]^{\left(\frac{1}{\alpha_{B,j_1}\alpha_{Be,j_2}-\alpha_{Be,j_1}\alpha_{B,j_2}}\right)}, \quad (27)$$

$$x_{j_2} = x^0_{j_2} \times \left[\left(\frac{\phi_B}{\phi_B^{SSM}}\right)^{\alpha_{Be,j_1}} \times \left(\frac{\phi_{Be}}{\phi_{Be}^{SSM}}\right)^{-\alpha_{B,j_1}}\right]^{\left(\frac{1}{\alpha_{B,j_2}\alpha_{Be,j_1}-\alpha_{Be,j_2}\alpha_{B,j_1}}\right)}, \quad (28)$$

and

$$\left(\frac{\Delta x_{j_1}}{x_{j_1}}\right) = \left|1-\exp\left[\frac{\left(\frac{1}{\alpha_{B,j_2}}\right)^2\Delta(\ln\phi_B)^2 + \left(\frac{1}{\alpha_{Be,j_2}}\right)^2\Delta(\ln\phi_{Be})^2 + \sum_{j\neq j_1,j_2}\left(\frac{\alpha_{B,j}}{\alpha_{B,j_2}} - \frac{\alpha_{Be,j}}{\alpha_{Be,j_2}}\right)^2\Delta(\ln x^0_j)^2}{\left(\frac{\alpha_{B,j_1}}{\alpha_{B,j_2}} - \frac{\alpha_{Be,j_1}}{\alpha_{Be,j_2}}\right)^2}\right]^{\frac{1}{2}}\right| \quad (29)$$

$$\left(\frac{\Delta x_{j_2}}{x_{j_2}}\right) = \left|1-\exp\left[\frac{\left(\frac{1}{\alpha_{B,j_1}}\right)^2\Delta(\ln\phi_B)^2 + \left(\frac{1}{\alpha_{Be,j_1}}\right)^2\Delta(\ln\phi_{Be})^2 + \sum_{j\neq j_1,j_2}\left(\frac{\alpha_{B,j}}{\alpha_{B,j_1}} - \frac{\alpha_{Be,j}}{\alpha_{Be,j_1}}\right)^2\Delta(\ln x^0_j)^2}{\left(\frac{\alpha_{B,j_2}}{\alpha_{B,j_1}} - \frac{\alpha_{Be,j_2}}{\alpha_{Be,j_1}}\right)^2}\right]^{\frac{1}{2}}\right| \quad (30)$$

Equations (27) and (28) imply, in particular, that the values of the parameters x_{j_1} and x_{j_2} could differ from their respective SSM values by factors determined by the four logarithmic derivatives - α_{B,j_1}, α_{B,j_2}, α_{Be,j_1} and α_{Be,j_2}. For calculating the SSM parameter uncertainties, we take all possible combinations of $\{x_{j_1},x_{j_2}\}$ and use eqs. (29) and (30) to get the corresponding errors on

combinations		$\frac{\Delta x_{j_1}}{x_{j_2}}$ (%)	$\frac{\Delta x_{j_2}}{x_{j_2}}$ (%)
j_1	j_2		
S_{34}	S_{11}	12.71	8.81
S_{34}	Z/X	14.25	12.26
S_{34}	$D_\odot$	13.79	11.35

Table 4: The list of different combinations of two SSM parameters which are determined with an uncertainty smaller than 15% using data of prospective direct measurements of ^8B and ^7Be neutrino fluxes with 1σ errors of 4% and 6%, respectively. The relative uncertainties on the SSM parameters thus determined are also given.

these sets of two parameters, assuming a measurement of ^8B and ^7Be neutrino fluxes respectively with 4% and 6% uncertainty [10]. The uncertainties of x_{j_1} and x_{j_2}, as given in eqs. (29) and (30), are controlled in a rather complicated way by both the magnitude and the relative signs of the different logarithmic derivatives. However, it follows from eqs. (29) and (30) that if for a certain pair of SSM parameters x_{j1} and x_{j2} the relation $\alpha_{B,j_2}/\alpha_{B,j_1} \cong \alpha_{Be,j_2}/\alpha_{Be,j_1}$ holds, these parameters would be determined with poor accuracy even if one uses high precision data on the ^8B and ^7Be neutrino fluxes. Table 2 suggests that such pairs can be, for instance, $\{S_{33}, S_{34}\}$, $\{L_\odot, O_\odot\}$ and $\{Z/X, D_\odot\}$.

Among the solar physics parameters the opacity $O_\odot$ can be determined with a 9% uncertainty in pair with S_{34}, or S_{17}, or S_{e^-7}, for which we get at the same time uncertainty of approximately 16%. For the diffusion $D_\odot$ we find an uncertainty of 11% when determined in combination with S_{33} or S_{34}. In these cases, however, S_{33} and S_{34} are found within 36% and 14%, respectively.

In Table 4 we present results only for those combinations of two SSM parameters which are determined with an uncertainty smaller than 15% using directly measured values of ϕ_B and ϕ_{Be}. It is interesting to note from Table 4 that the set $\{S_{34}, Z/X\}$ can be determined with a relatively good precision: the predicted uncertainty of Z/X is smaller than the estimated one within the BP04 SSM [13], while the uncertainty in S_{34} is approximately 14.3%. The latter should be compared with the 1σ error of 9.4%, quoted when S_{34} is obtained using the data on the reaction $^3He + ^4He \rightarrow ^7 Be + \gamma$. The parameter S_{34} can be determined with 12.7% uncertainty in the combination $\{S_{34}, S_{11}\}$. As can be seen by comparing Table 3 with Table 4, one gets a somewhat better precision on Z/X if the value of Z/X is obtained from the experimental information on the ^8B neutrino flux only [11]. However, the uncertainty in the determination of S_{34} can be much smaller when in addition to the data on the ^7Be neutrino flux one uses the data on the ^8B neutrino flux as well.

In Table 5 we present results on the uncertainties of the parameter combination $\{Z/X, S_{34}\}$, determined from data on the ^8B and ^7Be neutrino fluxes. The results correspond to 1σ errors of 2%, 3% and 4% in the measured value of the ^8B neutrino flux, and of 2%, 4%, 6%, 8% and 10% in the measured value of ϕ_{Be}. We observe that if the 1σ uncertainties in the values of ϕ_B and ϕ_{Be} are smaller than 4%, the uncertainty in the value of S_{34}, determined using the neutrino flux

[10] While ϕ_B is already known with a 4% uncertainty, ϕ_{Be} could be determined, as we have discussed above, with a 6% error by the Borexino experiment (see ref. [10] for details).

[11] The reason for this behavior can be traced to the complicated nature of the eqs. (29) and (30).

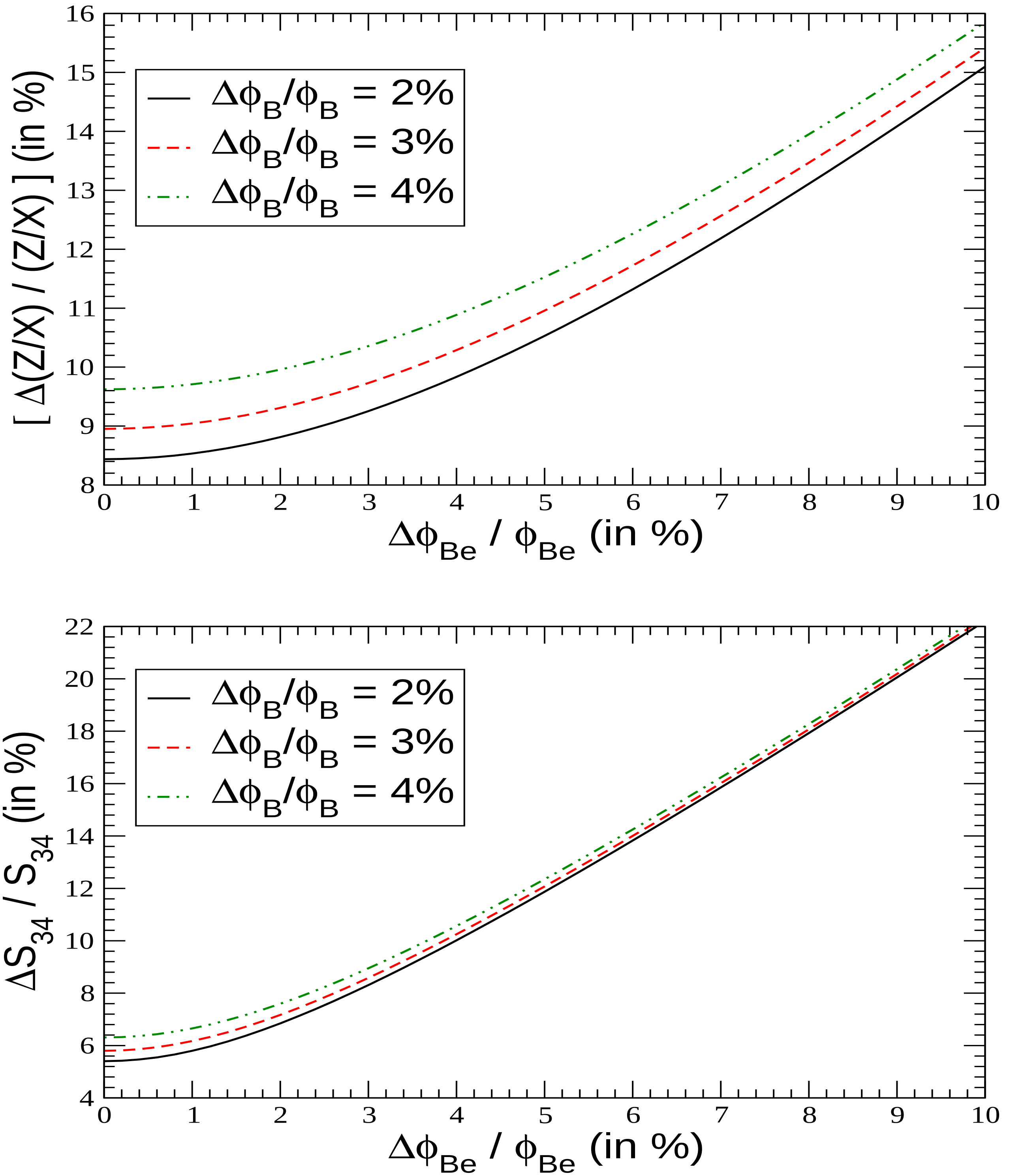

Figure 3: The 1σ fractional uncertainties of the SSM input parameters S_{34} and Z/X, determined from data on the ^8B and ^7Be neutrino fluxes, as a function of the uncertainty in the measured ^7Be neutrino flux, for three different values of the 1σ error in the measured ^8B neutrino flux.

$\frac{\Delta\phi_B}{\phi_B}$ (%)	$\frac{\Delta\phi_{Be}}{\phi_{Be}}$(%)	$\frac{\Delta(Z/X)}{(Z/X)}$ (%)	$\frac{\Delta S34}{S_{34}}$ (%)
2	2	8.81	6.85
	4	9.83	10.02
	6	11.32	13.83
	8	13.11	17.93
	10	15.10	22.21
3	2	9.30	7.16
	4	10.29	10.24
	6	11.72	14.00
	8	13.47	18.07
	10	15.42	22.34
4	2	9.96	7.60
	4	10.88	10.56
	6	12.26	14.25
	8	13.95	18.27
	10	15.85	22.51

Table 5: Relative uncertainties of the set of two parameters - Z/X and S_{34}, determined using prospective high precision data on the ^8B and ^7Be neutrino fluxes. The results correspond to different sets of assumed 1σ errors in the measured values of the two fluxes.

measurements, would be smaller than its present uncertainty of $\sim 9\%$. The uncertainty in the determination of Z/X would be smaller than its currently SSM estimated one of 15% if ϕ_{Be} is measured with an error not exceeding approximately 10% at 1σ.

In Fig. 3 we show the expected 1σ uncertainties of the pair of parameters $\{S_{34}, Z/X\}$, $\Delta S_{34}/S_{34}$ and $\Delta(Z/X)/(Z/X)$, as continuous functions of the uncertainty in the measured ^7Be neutrino flux, $\Delta\phi_{Be}/\phi_{Be}$, for three different values for the uncertainty in the measured ^8B neutrino flux, $\Delta\phi_B/\phi_B$. The figure shows that the dependence of $\Delta S_{34}/S_{34}$ on $\Delta\phi_{Be}/\phi_{Be}$ is stronger than that on $\Delta\phi_B/\phi_B$, while $\Delta(Z/X)/(Z/X)$ depends on the accuracy of measurement of both fluxes. This feature is due to the fact that $\alpha_{Be,Z/X} \simeq 2.2 \times \alpha_{B,Z/X}$, while $\alpha_{Be,S_{34}} \simeq \alpha_{B,S_{34}}$.

7 Determining Three SSM Input Parameters Using Data on the ^8B, ^7Be and pp Neutrino Fluxes

As we have already discussed in Section 3, the pp neutrino flux, ϕ_{pp}, is determined with an uncertainty of 2% from the present solar and reactor neutrino data if one employs the luminosity constraint. Future solar neutrino experiments can provide a remarkably precise measurement of

ϕ_{pp} [10]. In this Section we use high precision prospective data on ^8B , ^7Be and pp neutrino fluxes to simultaneously determine any three of the SSM input parameters.

Equations (19) and (24) for K=3 give the central values and uncertainties for any three parameters x_{j_1}, x_{j_2} and x_{j_3} as

$$x_r = \left[\left(\frac{\phi_{pp}}{\phi_{pp}^{SSM}}\right)^{\beta_{pp,r}} \left(\frac{\phi_{Be}}{\phi_{Be}^{SSM}}\right)^{\beta_{Be,r}} \left(\frac{\phi_{B}}{\phi_{B}^{SSM}}\right)^{\beta_{B,r}}\right]^{1/DetA} \times x_r^0, \qquad (r = j_1, j_2, j_3) \tag{31}$$

and

$$\left[\ln\left(1+\frac{\Delta x_r}{x_r}\right)\right]^2 = \\ \left(\frac{1}{\mathrm{DetA}}\right)^2 \times \left[\beta_{pp,r}^2 \left\{\ln\left(1+\frac{\Delta\phi_{pp}}{\phi_{pp}}\right)\right\}^2 + \beta_{Be,r}^2 \left\{\ln\left(1+\frac{\Delta\phi_{Be}}{\phi_{Be}}\right)\right\}^2 + \beta_{B,r}^2 \left\{\ln\left(1+\frac{\Delta\phi_{B}}{\phi_{B}}\right)\right\}^2 \right. \\ \left. + \sum_{j\neq j_1,j_2,j_3} (\beta_{pp,r}\alpha_{pp,j} + \beta_{Be,r}\alpha_{Be,j} + \beta_{B,r}\alpha_{B,j})^2 \left\{\ln\left(1+\frac{\Delta x_j}{x_j}\right)\right\}^2\right], \quad (r = j_1, j_2, j_3) \tag{32}$$

where A is 3×3 matrix given by

$$A = \begin{pmatrix} \alpha_{pp,j_1} & \alpha_{pp,j_2} & \alpha_{pp,j_3} \\ \alpha_{Be,j_1} & \alpha_{Be,j_2} & \alpha_{Be,j_3} \\ \alpha_{B,j_1} & \alpha_{B,j_2} & \alpha_{B,j_3} \end{pmatrix}. \tag{33}$$

$\beta_{pp,r}$, $\beta_{Be,r}$ and $\beta_{B,r}$ are the cofactors of the matrix elements $\alpha_{pp,r}$, $\alpha_{Be,r}$ and $\alpha_{B,r}$ respectively ($r = j_1, j_2, j_3$). We take different combinations of three SSM parameters and calculate their uncertainties using 3%, 4% and 1% as illustrative 1σ errors in the measured values of ^8B, ^7Be and pp neutrino fluxes, respectively. We find that for almost all solar model parameters, the uncertainties reduce when using the combined information on the ^7B, ^8Be and pp neutrino fluxes, compared to what we have obtained by using only prospective data on ^8B and/or ^7Be fluxes. In Table 6 we present results only for those sets of three SSM input parameters which are determined with uncertainties smaller than 15% each.

As Table 6 shows, the most precise determination of S_{34} occurs in the combination $\{S_{34}, L_\odot, S_{e^-7}\}$, while Z/X is best determined in the sets $\{S_{17}, L_\odot, Z/X\}$ and $\{S_{e^-7}, L_\odot, Z/X\}$. For both these parameters the uncertainties we get are smaller than those in the respective BP04 SSM predictions.

Note also that we get a rather accurate determination of S_{17} from the combination $\{S_{34}, S_{17}, L_\odot\}$, and of $L_\odot$ from $\{S_{34}, L_\odot, Z/X\}$. Although the uncertainty on S_{17} thus obtained of $\sim$(8% - 10%) is larger than the currently estimated uncertainty in the value of S_{17} found from relevant nuclear reaction data (see Section 2), our results on S_{17} can be used, in particular, as a consistency check, e.g., of the extrapolation procedure employed to get S_{17} from the data. In what concerns the other parameters, we get the best determination of S_{11} from $\{S_{11},, L_\odot, S_{e^-7}\}$, of $D_\odot$ from $\{S_{17}, L_\odot, D_\odot\}$, and of $\tau_\odot$ from $\{S_{17}, L_\odot, \tau_\odot\}$.

We stress that even though the precision we get for $L_\odot$ is worse than the precision achieved in the direct measurement of $L_\odot$, the method we used to determine $L_\odot$ allows to perform a

combinations			$\frac{\Delta x_{j_1}}{x_{j_2}}$ (%)	$\frac{\Delta x_{j_2}}{x_{j_2}}$ (%)	$\frac{\Delta x_{j_3}}{x_{j_3}}$ (%)
x_{j_1}	x_{j_2}	x_{j_3}			
S_{11}	S_{34}	$L_\odot$	6.16	7.98	1.62
S_{11}	$L_\odot$	$O_\odot$	13.67	1.72	12.25
S_{34}	$L_\odot$	Z/X	8.87	1.00	8.63
S_{34}	$L_\odot$	$O_\odot$	10.45	1.69	5.31
S_{34}	$L_\odot$	$D_\odot$	9.03	1.57	5.77
S_{34}	$L_\odot$	$\tau_\odot$	11.40	1.66	12.86
S_{11}	S_{17}	$L_\odot$	4.73	11.05	1.73
S_{11}	$L_\odot$	S_{e^-7}	4.53	1.73	11.05
S_{33}	S_{17}	$L_\odot$	12.40	8.47	1.95
S_{33}	$L_\odot$	S_{e^-7}	13.43	1.95	8.47
S_{34}	S_{17}	$L_\odot$	8.61	8.98	1.86
S_{34}	$L_\odot$	S_{e^-7}	6.33	1.85	8.98
S_{17}	$L_\odot$	Z/X	9.26	1.21	6.65
S_{17}	$L_\odot$	$\tau_\odot$	9.05	1.85	7.34
S_{17}	$L_\odot$	$O_\odot$	9.21	1.84	3.40
S_{17}	$L_\odot$	$D_\odot$	9.74	1.74	4.68
$L_\odot$	Z/X	S_{e^-7}	1.21	6.65	9.88
$L_\odot$	$\tau_\odot$	S_{e^-7}	1.85	7.42	9.29
$L_\odot$	$O_\odot$	S_{e^-7}	1.84	3.40	9.48
$L_\odot$	$D_\odot$	S_{e^-7}	1.73	4.68	10.12

Table 6: The fractional uncertainties in different combinations of the SSM input parameters for which we get uncertainty smaller than 15%, assuming 3%, 4% and 1% errors in the "measured" ϕ_B, ϕ_{Be} and ϕ_{pp} neutrino fluxes, respectively.

fundamental test of the thermo-nuclear fusion theory of energy generation in the Sun, as well as to test the hypothesis that the Sun is in an approximate steady state in what regards the energy produced in its central region and the energy emitted from its surface.

8 Conclusions

In the present article we have studied the possibility of using the precision data (current and prospective) on the i) ^{8}B, ii) ^{8}B and ^{7}Be, and iii) ^{8}B, ^{7}Be and *pp*, solar neutrino fluxes in order to obtain "direct" information (i.e., to constrain or determine) on at least some of the eleven solar model parameters - opacity, diffusion, heavy element surface abundance, nuclear reaction S-factors, etc., which enter into the calculations of the fluxes in the Standard Solar Model (SSM). Our work was inspired by the remarkable progress made in the studies of solar neutrinos in the last several years, which led to an unexpectedly precise determination of the solar neutrino oscillation parameters and of the ^{8}B neutrino flux, as well as by the prospects for high precision measurements of the ^{7}Be and pp neutrino fluxes. It was stimulated also by the realization that the solar physics parameters like the opacity ($O_\odot$), diffusion ($D_\odot$) and heavy element surface abundance (Z/X), can never be measured in direct experiments. The solar photon luminosity $L_\odot$ is measured directly with very high accuracy. However, the "conventionally" measured luminosity of the Sun is determined by photons produced in the central region of the Sun, which took $\sim 4\times10^4$ years to reach the surface of the Sun from which they are emitted (see, e.g., [14]). The luminosity determined from solar neutrino flux measurements provides "real time" information on the rates of nuclear fusion reactions in the central region of the Sun, in which the solar energy is generated: neutrinos are simultaneously produced in these reactions with the photons observed in the form of solar luminosity, but it takes solar neutrinos approximately only 8 minutes to reach the Earth. Similar considerations apply, perhaps to somewhat less extent, to the S-factors S_{11}, S_{33}, S_{34}, $S_{1,14}$ and S_{17}, directly related to the rates of the nuclear fusion reactions, on which the SSM predictions for the solar neutrino fluxes depend and in which the solar energy is generated. They can be and are measured in direct experiments on Earth. However, this is done at energies which are considerably higher than the energies at which the reactions take place in the central part of the Sun. As a consequence, one has to employ an extrapolation procedure (based on nuclear theory) in order to obtain the values of the rates at the energy of interest, corresponding to the physical conditions in the central part of the Sun.

We have derived the basic equations for determining the central values of the SSM input parameters and their uncertainties using results of direct measurements of solar neutrino fluxes (Section 4, eqs. (19) and (24)). If we have r measured solar neutrino fluxes, at most r SSM input parameters can be determined using the data on the solar neutrino fluxes. For the remaining SSM parameters we have to use the SSM values and estimated uncertainties. All our numerical results are based on the predictions of the SSM of Bahcall and Pinsonneault from 2004 [13]. These include the dependence of different solar neutrino fluxes (^8B , ^7Be , pp) on the SSM input parameters, and, whenever necessary, the predicted values of the SSM input parameters and their uncertainties.

We used first the precise value of the ^8B neutrino flux, ϕ_B, obtained from global analysis of solar neutrino and KamLAND data, to determine each of the SSM parameters on which ϕ_B depends. If the measured mean value of ϕ_B differs from the value predicted by the SSM, ϕ_B^{SSM}, the value of the parameter x_{j_1} obtained using the data on ϕ_B would differ from its SSM predicted value $x^0_{j_1}$ by a factor controlled by the logarithmic derivative $\alpha_{B,j_1} = \frac{\partial \ln \phi_B}{\partial \ln x_{j_1}}$ (see eq. (25)). The relative uncertainty in the parameter x_{j_1} thus found depends on the inverse power of the magnitude of α_{B,j_1} (eq. (26)). Since, according to the BP04 SSM, the relevant logarithmic derivatives in the cases of the nuclear reaction S-factors S_{33} and $S_{1,14}$ are relatively small (see Table 2), these quantities cannot be determined with sufficiently good accuracy even by using a high precision measurement of the ^{8}B neutrino flux. The results of this part of our analysis are summarised in Table 3. We have found, in particular, that the SSM parameters like S_{11}, Z/X, $L_\odot$ and $O_\odot$ can be determined with uncertainties less than 10% owing to the relatively large values of their corresponding logarithmic derivatives. For the uncertainty in diffusion parameter $D_\odot$ we get approximately 10.6%. Our results show that the parameter Z/X can be determined with an uncertainty which is smaller than its currently estimated uncertainty in the BP04 SSM [13]. We have found also that the uncertainties of most of the SSM parameters under discussion practically do not change when the

1σ error in the measured ^8B neutrino flux is reduced from 4% to 2%.

We have performed a similar analysis by combining a prospective high precision measurement of the ^7Be neutrino flux with the ^8B neutrino flux measurement. In this case it is possible to determine simultaneously two SSM input parameters, x_{j_1} and x_{j_2}, including their uncertainties, using the data on the ^8B and ^7Be neutrino fluxes. Our results show, in particular, that the values of the parameters x_{j_1} and x_{j_2} thus determined could differ from their respective SSM values by factors determined by the four logarithmic derivatives - α_{B,j_1}, α_{B,j_2}, α_{Be,j_1} and α_{Be,j_2} (eqs. (27) and (28)). We have calculated the SSM parameter uncertainties for all possible combinations of two SSM parameters $\{x_{j_1},x_{j_2}\}$, assuming that ^8B and ^7Be neutrino fluxes are measured with 1σ errors of 4% and 6%, respectively. Such precision on ϕ_{Be} can be reached in the Borexino experiment. We have found that the uncertainties of x_{j_1} and x_{j_2} are controlled in a rather complicated way by both the magnitude and the relative signs of the different logarithmic derivatives (eqs. (29) and (30)). If for a certain pair of SSM parameters x_{j1} and x_{j2} the relation $\alpha_{B,j_2}/\alpha_{B,j_1} \cong \alpha_{Be,j_2}/\alpha_{Be,j_1}$ holds, these parameters would be determined with poor accuracy even if one uses high precision data on the ^8B and ^7Be neutrino fluxes. The logarithmic derivatives taken from the BP04 SSM (table 2) suggest that such pairs can be, for instance, $\{S_{33}, S_{34}\}$, $\{L_\odot, O_\odot\}$ and $\{Z/X, D_\odot\}$. We have found also that among the solar physics parameters the opacity $O_\odot$ can be determined with a 9% uncertainty in pair with S_{34}, or S_{17}, or S_{e^-7}, for which we get at the same time uncertainty of approximately 16%. For the diffusion $D_\odot$ we find an uncertainty of 11% when determined in combination with S_{33} or S_{34}. In these cases the latter are found with uncertainties of 36% and 14%, respectively. Most of the results from this part of our study are summarised in Table 4.

We have obtained also rather detailed results on the uncertainties of the combination $\{Z/X,S_{34}\}$, determined from data on the ^8B and ^7Be neutrino fluxes (Table 5 and Fig. 3). We have found, in particular, that if the 1σ uncertainties in the values of ϕ_B and ϕ_{Be} are smaller than 4%, the uncertainty in the value of S_{34}, determined using the neutrino flux measurements, would be smaller than its presently estimated uncertainty of $\sim 9\%$. The uncertainty in the determination of Z/X would be smaller than its currently SSM estimated one of 15% if ϕ_{Be} is measured with an error not exceeding approximately 10% at 1σ.

Finally, we have analyzed the possibility to use high precision prospective measurements of the ^8B , ^7Be and pp solar neutrino fluxes to simultaneously determine any three of the SSM input parameters. We have taken different combinations of three SSM parameters and calculate their uncertainties using 3%, 4% and 1% as illustrative 1σ errors in the measured values of ϕ_B, ϕ_{Be} and ϕ_{pp}, respectively. We have found that for almost all solar model parameters, the uncertainties reduce when using the combined information on the ^7B, ^8Be and pp neutrino fluxes, compared to what we have obtained by using only prospective data on ^8B and/or ^7Be fluxes. Results for those sets of three SSM input parameters which are determined with uncertainties smaller than 15% each are collected in Table 6. Our results show, in particular, that the most precise determination of S_{34} occurs in the combination $\{S_{34}, L_\odot, S_{e^-7}\}$, while Z/X is best determined in the set $\{S_{17}, L_\odot, Z/X\}$. For both these parameters the uncertainties we get are smaller than those in the respective BP04 SSM predictions. The best determination of S_{11} is found to be from the set $\{S_{11}, , L_\odot, S_{e^-7}\}$, of $D_\odot$ from $\{S_{17}, L_\odot, D_\odot\}$, and of $\tau_\odot$ from $\{S_{17}, L_\odot, \tau_\odot\}$. Even though the precision we obtained for $L_\odot$ is worse than the precision achieved in the direct measurement of $L_\odot$, the method used to determine $L_\odot$ allows to perform a fundamental test of the thermo-nuclear fusion theory of energy generation in the Sun, as well as to test the hypothesis that the Sun is in an approximate steady state in what regards the energy produced in its central region and the energy emitted from its surface.

The results obtained in the present article underline the importance of performing high precision measurements of ^7Be and pp solar neutrino fluxes.

Acknowledgments. We dedicate this paper to the memories of J.N. Bahcall and R. Davis. S.C. and S.T.P. would like to thank S.M. Bilenky for useful discussions. The collaboration of T. Schwetz at the initial stage of this study is acknowledged with gratefulness. This work was supported in part by the Italian MIUR and INFN under the programs "Fisica Astroparticellare" (S.T.P.). The work of S.G. was supported by the Alexander–von–Humboldt–Foundation. The work of A.B. is partly supported by the Partner Group program between the Max Planck Institute for Physics and Tata Institute of Fundamental Research.

References

[1] B. T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998); Y. Fukuda *et al.* [Kamiokande Collaboration], Phys. Rev. Lett. **77**, 1683 (1996); J. N. Abdurashitov *et al.* [SAGE Collaboration], J. Exp. Theor. Phys. **95**, 181 (2002); T. Kirsten *et al.* [GALLEX and GNO Collaborations], Nucl. Phys. B (Proc. Suppl.) **118**, 33 (2003); C. Cattadori, Talk given at ν'04 International Conference, June 14-19, 2004, Paris, France.

[2] S. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Lett. **B539**, 179 (2002).

[3] Q. R. Ahmad *et al.* [SNO Collaboration], Phys. Rev. Lett. **87**, 071301 (2001); *ibid.* **89**, 011301 and 011302 (2002); S. N. Ahmed *et al.*, Phys. Rev. Lett. **92**, 181301 (2004).

[4] K. Eguchi *et al.* [KamLAND Collaboration], Phys. Rev. Lett. **90**, 021802 (2003).

[5] B. Pontecorvo, Zh. Eksp. Teor. Fiz. (JETP) **33**, 549 (1957) and **34**, 247 (1958).

[6] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **53**, 1717 (1967) [Sov. Phys. JETP **26**, 984 (1968)].

[7] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).

[8] S.T. Petcov, Lecture Notes in Physics, v. 512 (eds. H. Gausterer and C.B. Lang, Springer, 1998), p. 281 (hep-ph/9806466);
Proceedings of the 6th and 7th ICTP Schools on Non–Accelerator Particle Astrophysics, 9 - 20 July, 2001, and 26 July - 6 August, 2004, Miramare-Trieste, Italy, (eds. R.A. Carrigan *et al.*, World Scientific, Singapore, 2002 and 2005), p. 14 and p. 21.

[9] B. Aharmim *et al.* [SNO Collaboration], Phys. Rev. C **72**, 055502 (2005).

[10] J. N. Bahcall and C. Pena-Garay, JHEP **0311**, 004 (2003).

[11] A. Bandyopadhyay *et al.*, Phys. Lett. B **608**, 115 (2005); S. Choubey, arXiv:hep-ph/0509217; S. Goswami, Int. J. Mod. Phys. A **21**, 1901 (2006); see also A. Bandyopadhyay *et al.*, Phys. Lett. B **583**, 134 (2004).

[12] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, Prog. Part. Nucl. Phys. **57**, 71 (2006); T. Schwetz, hep-ph/0606060; J. N. Bahcall, M. C. Gonzalez-Garcia and C. Pena-Garay, JHEP **0408**, 016 (2004).

[13] J. N. Bahcall and M. H. Pinsonneault, Phys. Rev. Lett. **92**, 121301 (2004).

[14] J. N. Bahcall and C. Pena-Garay, New J. Phys. **6**, 63 (2004).

[15] J. N. Bahcall and A. M. Serenelli, Astrophys. J. **626** (2005) 530.

[16] J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys. **64** (1992) 85.

[17] J. N. Bahcall, S. Basu and M. H. Pinsonneault, Astrophys. J. **555** (2001) 990.

[18] A. R. Junghans *et al.*, Phys. Rev. C **68**, 065803 (2003) and references therein.

[19] T. Motobayashi, Nucl. Phys. A **693** (2001) 258; References [16] to [20] in [18].

[20] L. Trache *et al.*, Phys. Rev. C **67**, 062801 (2003) and earlier references there.

[21] G. Tabacaru *et al.*, Phys. Rev. C **73**, 025808 (2006).

[22] E. G. Adelberger *et al.*, Rev. Mod. Phys. **70**, 1265 (1998).

[23] M. Gai, arXiv:nucl-ex/0510081.

[24] B.S. Singh *et al.*, nucl-ex/0407017.

[25] R. Menagazzo (LUNA collaboration), Talk at Neutrino 2006, Santa-Fe, USA; http://neutrinosantafe06.com .

[26] C. Allende Prieto, D.L. Lambert and M. Asplund, Astrophys. J. **573** (2002) L137; M. Asplund, N. Grevesse and A.J. Sauval, astro-ph/0410214.

[27] N. Grevesse and A.J. Sauval, Space. Sci. Rev. **70** (1998) 161.

[28] J. N. Bahcall, A. M. Serenelli and S. Basu, Astrophys. J. **621**, L85 (2005).

[29] S. Choubey and S.T. Petcov, Phys. Lett. B **594**, 333 (2004).

[30] A. Bandyopadhyay *et al.*, Phys. Rev. D **72**, 072002 (2005).

[31] J. Beacom and M. Vagins, Phys. Rev. Lett. **93**, 171101 (2004).

[32] A. Bandyopadhyay, S. Choubey and S. Goswami, Phys. Rev. D **67**, 113011 (2003).

[33] S. T. Petcov and T. Schwetz, hep-ph/0607155.

[34] A. Bandyopadhyay *et al.*, Phys. Lett. B **581**, 62 (2004).

[35] H. Minakata *et al.*, Phys. Rev. D **71**, 013005 (2005).

[36] J. F. Kopp *et al.*, hep-ph/0606151.

[37] A. Poon, talk at Lepton-Photon 2005, Uppsala, http://lp2005.tsl.uu.se∼lp2005.

[38] S. Schönert, talk at Neutrino 2002, Munich, Germany, (*http://neutrino2002.ph.tum.de*).

[39] M. Spiro and D. Vignaud, Phys. Lett. B **242** (1990) 279; J. N. Bahcall, Phys. Rev. C **65**, 025801 (2002).

[40] C. Galbiati, talk at Neutrino 2004, Paris, http://neutrino2004.in2p3.fr.

[41] T. Mitsui, talk at Neutrino 2006, Santa Fe, http://neutrinosantafe06.com.