

Supersymmetry breaking at the end of a cascade of Seiberg dualities

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Abstract

We study the IR dynamics of the cascading non-conformal quiver theory on N regular and M fractional D3 branes at the tip of the complex cone over the first del Pezzo surface. The horizon of this cone is the irregular Sasaki-Einstein manifold $Y^{2,1}$. Our analysis shows that at the end of the cascade supersymmetry is dynamically broken.

1 Introduction and main results

Recently, a new class of AdS/CFT dual pairs has been found [1, 2, 3] and a number of duality checks have been successfully performed [4, 3]. The ten-dimensional type IIB supergravity background has $AdS_5 \times Y^{p,q}$ geometry, constant F_5 -flux through $Y^{p,q}$ and constant dilaton and axion. $Y^{p,q}$ are five-dimensional Sasaki-Einstein manifolds, where $q < p$ are positive coprime integers.

The dual gauge theories are four dimensional $\mathcal{N} = 1$ SCFT's describing the low energy dynamics of a bunch of N D3-branes placed at the tip of the Calabi-Yau cone C over $Y^{p,q}$. The moduli space of the theory is described by N copies of the defining equations of the cone. Since the topology of the $Y^{p,q}$ manifolds is $S^3 \times S^2$ one can consider the possibility of adding fractional branes to this system. The latter can be thought of as D5-branes wrapped on S^2 . From the gauge theory point of view this breaks conformal invariance. From the supergravity point of view this changes the geometry drastically. In particular, the geometry of the cone is expected to be deformed since in general a deformation should occur in the field theory moduli space. Moreover, the complex type IIB three-form $G_3 = F_3 + \tau H_3$ acquires a non-trivial profile and the RR five-form flux a radial dependence.

A first crucial step towards finding the complete supergravity solution corresponding to a bunch of N regular and M fractional D3-branes has been taken in [5]. As a simplifying assumption, the geometry of the cone was taken unchanged with respect to the conformal case. The results in [5] match with the dual field theory expectations in the UV, in the limit $M \ll N$. Still, the solution has a singularity in the region corresponding to the IR of the dual field theory. Showing the existence of a smooth deformation of the geometry remains an open problem. Actually, there are geometrical arguments against the existence of complex deformations of the cone $C(Y^{p,q})$ preserving supersymmetry [6], though in [7] a first order approximation to such a deformation has been found.

The structure of the solution found in [5], in particular the fact that the F_5 -flux is not constant, suggests that duality cascade phenomena might take place as the theory flows through the IR, in analogy to the conifold case [8, 9]. For the cases $Y^{p,p-1}$ and $Y^{p,1}$, the cascade can reach a point where the gauge group becomes $SU(M) \times SU(2M) \times \dots \times SU(2pM)$ and for the $SU(2pM)$ factor there are effectively $2pM$ flavors. This implies that the moduli space of the theory will receive quantum corrections and so a deformation of the Calabi-Yau cone should occur.

In this paper we try to understand this deformation from a dual perspective. We

analyze, from a pure field theory point of view, the case of the theory for $Y^{2,1}$ (in this case the Calabi-Yau is the complex cone over the first del Pezzo surface dP_1), which can be seen as a master example for both $Y^{p,p-1}$ and $Y^{p,1}$ cases. Our results differ sensibly from what happens for the conifold. Most notably, the theory does not have a supersymmetric vacuum. Therefore, a dual smooth supergravity background, if it exists, should correspond to a non-supersymmetric deformation of the singular geometry. This result, which is likely to hold for the full $Y^{p,p-1}$ and $Y^{p,1}$ series, opens up the very interesting possibility of dealing with an all new class of non-*AdS*/non-CFT dual pairs with broken supersymmetry, where a number of non-supersymmetric field theory IR phenomena could be tackled in a well defined setting.

This note is organized as follows. In section 2 we briefly review the structure of the gauge theory for dP_1 recalling how the cascade of Seiberg dualities takes place. In section 3, which contains our main results, we follow the cascade step by step up to the point where quantum deformations of the moduli space are expected. We thus study in detail this point and show that supersymmetry is dynamically broken.

Note added: While this paper was being completed, two works appeared [12, 13] which address similar issues. Their conclusions agree with ours.

2 Regular and fractional D3-branes on $C(Y^{2,1})$

Let us place N D3-branes at the tip of $C(Y^{2,1})$, which is the complex cone over the first del Pezzo surface dP_1 [2]. The conformal field theory has gauge group $SU(N)^4$, bi-fundamental matter and a marginal superpotential [10]. The *AdS*/CFT correspondence was checked for this case in [4] where the exact R-charges and the central charge of the theory were computed using a-maximization and shown to agree exactly with the dual supergravity predictions made in [2].

The addition of M fractional D3-branes (i.e. D5-branes wrapped on S^2) was considered in [11]. The corresponding quiver diagram is reported in figure 1. There is also a superpotential

$$W = \text{Tr} \left[\epsilon_{\alpha\beta} X_{34}^\alpha X_{41}^\beta X_{13} - \epsilon_{\alpha\beta} X_{34}^\alpha X_{42} X_{23}^\beta + \epsilon_{\alpha\beta} X_{34}^3 X_{41}^\alpha X_{12} X_{23}^\beta \right], \quad (1)$$

which breaks the global symmetry group to $SU(2) \times U(1) \times U(1)$. The chiral fields X_{41}^α , X_{34}^α , X_{23}^α are doublets with respect to the $SU(2)$ symmetry

Let us consider some of the properties of this non conformal theory, beginning with the β -functions. We will take $M \ll N$ in the following. For any node the β -function is

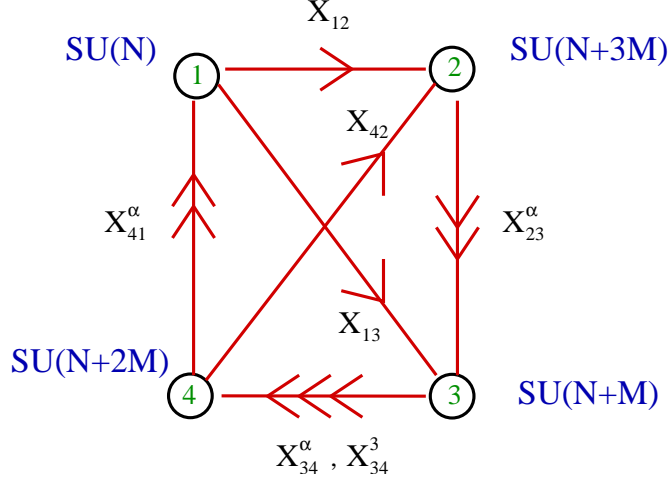


Figure 1: The non conformal quiver associated to the first del Pezzo surface. Each node represents a gauge factor. Each arrow represents a bi-fundamental chiral multiplet. The chiral fields X_{41}^α , X_{34}^α , X_{23}^α are doublets with respect to the $SU(2)$ flavor symmetry. The quiver for the conformal case is just the same, with $M = 0$.

proportional to

$$b \equiv [3T(G) - \sum_i T(r_i)(1 - \gamma_i)], \quad (2)$$

where γ_i are the conformal dimensions of the fields and $T(G)$, $T(r_i)$ are the Casimir of the adjoint and r_i representations.

The anomalous dimensions γ_i of the bi-fundamental fields in the theory are expected to differ from the ones at the conformal fixed point ($M = 0$ case) by factors of order $(M/N)^2$. This is because the quiver in figure 1 is invariant under $N \rightarrow N + 3M$, $M \rightarrow -M$. In the limit $M \ll N$ this corresponds to $N \rightarrow N$, $M \rightarrow -M$, hence the anomalous dimensions cannot depend linearly on M/N . This means that in order to give the β -functions for each node of the quiver at leading order in M/N , we will only need to know the anomalous dimensions (or the exact R-charges $3R_i = \gamma_i + 2$) for the conformal case.

Using the fact that the β -functions are zero in the conformal case, we end up with the following results

$$\begin{aligned} b_1 &= \frac{3M}{2}[4(R_{14} - 1) + (R_{13} - 1) + 3(R_{12} - 1)], \\ b_2 &= 3M[3 + (R_{23} - 1) + (R_{24} - 1)], \\ b_3 &= 3M[1 + 3(R_{23} - 1) + 2(R_{34}^{(\alpha)} - 1) + (R_{34}^{(3)} - 1)], \\ b_4 &= \frac{3M}{2}[4 + 3(R_{24} - 1) + 2(R_{34}^{(\alpha)} - 1) + (R_{34}^{(3)} - 1)], \end{aligned} \quad (3)$$

where R_{ij} are the exact R-charges of the bi-fundamentals X_{ij} in the $M = 0$ case [4]

$$\begin{aligned} R(X_{12}) &= \frac{1}{3}(-17 + 5\sqrt{13}), & R(X_{23}^\alpha) &= R(X_{41}^\alpha) = \frac{4}{3}(4 - \sqrt{13}) \\ R(X_{34}^\alpha) &= \frac{1}{3}(-1 + \sqrt{13}), & R(X_{34}^3) &= R(X_{13}) = R(X_{42}) = -3 + \sqrt{13} \end{aligned} \quad (4)$$

Thus we find

$$\begin{aligned} b_1 &= -M(10 - \sqrt{13}), & b_2 &= -b_1 > 0, \\ b_3 &= -M(7\sqrt{13} - 22), & b_4 &= -b_3 > 0. \end{aligned} \quad (5)$$

There are couples of couplings running in opposite directions. Node (2) (whose gauge factor is $SU(N + 3M)$) will generically run to infinite coupling first. To follow the theory at smaller energy scales one can Seiberg dualize on this node, and proceed. Actually there will be a cascade of Seiberg dualities in which the number of colors will get smaller and smaller values, but the structure of the quiver and the superpotential will remain unchanged (until some node has $N_f = N_c$). A throughout analysis of how this occurs will be made in section 3.1.

All we have discussed in this section can be generalized to the cases $Y^{p,p-1}$ and $Y^{p,1}$, for which a similar dynamics takes place.

3 The duality cascade

In the cases $Y^{p,p-1}$, $Y^{p,1}$ the cascade of Seiberg dualities evolves to a theory with gauge group $SU(M) \times SU(2M) \times \dots \times SU(2pM)$ [5]. The last node has $N_f = N_c = 2pM$ and hence a non-perturbative modification of the moduli space of the theory will occur. Before this point is reached all the nodes have $N_f > N_c$ and the moduli space should not differ from the original one, that is (copies of) the cone over $Y^{p,q}$.

A counting of the number of flavors in the $Y^{2,1}$ case is useful at this point. If we consider the original theory, we find that the nodes of the $SU(N) \times SU(N + 3M) \times SU(N + M) \times SU(N + 2M)$ quiver have the following number of fundamental and anti-fundamental fields, respectively

$$N_f^{(1)} = 4M + 2N, \quad N_f^{(2)} = 2M + 2N, \quad N_f^{(3)} = 3N + 6M, \quad N_f^{(4)} = 3N + 3M. \quad (6)$$

Thus, starting with N being a multiple of M , as it decreases and eventually reaches $N = M$, we have that node (2) ends up with gauge group $SU(4M)$ and $N_f = N_c = 4M$. Hence one expects the moduli space to be modified.

Ignoring this fact, the last step of the cascade would reduce the number of gauge groups to three, giving a $SU(M) \times SU(2M) \times SU(3M)$ theory (i.e. node (1) has disappeared, see figure 1). The gauge group $SU(3M)$ would have $N_f = 2M < N_c$ and thus the related gauge theory (if one forgets the superpotential induced by the previous step) would not be expected to have a supersymmetric vacuum. As we will show in the following, a careful analysis of the cascade does not change this conclusion, though the physical picture is slightly different.

3.1 The self-similarity of the superpotential

It is known that the complex cone over dP_1 has only one toric phase [10]. This corresponds to the fact that (modulo rotations of the indexes) there is only one conformal field theory model corresponding to dP_1 . This means that the theory on dP_1 is self-similar, and this property is expected to be preserved in the non-conformal case as far as the Seiberg dual theory does not have nodes with N_f less or equal to N_c . Hence, at each step of the cascade, the quiver diagram looks the same and similarly the superpotential, up to the value of the superpotential coupling λ . For completeness, we will now prove the latter fact explicitly.

Let us consider first the conformal case $M = 0$. The superpotential is formally the same as in (1).¹ Let us consider the effect of a Seiberg duality on node (2). Since $N_f^{(2)} = 2N$ the duality gives a new node (2) with $N_c = N$, as before, and equal number of flavors. The new node (2) will have anti-fundamental fields x_{32}, y_{32} and fundamentals x_{21}, x_{24} . Moreover there will be a meson matrix which, in terms of the old variables will have block components $M_{13}^X = X_{12}X_{23}$, $M_{13}^Y = X_{12}Y_{23}$, $M_{43}^X = X_{42}X_{23}$, $M_{43}^Y = X_{42}Y_{23}$.

The superpotential inherited from the original theory will read

$$\begin{aligned}
W = & X_{34}Y_{41}X_{13} - Y_{34}X_{41}X_{13} - X_{34}M_{43}^Y + Y_{34}M_{43}^X + \\
& Z_{34}X_{41}M_{13}^Y - Z_{34}Y_{41}M_{13}^X.
\end{aligned}
\tag{7}$$

Moreover there will be a superpotential term (we redefine the fields so that the dimensional parameter $1/\mu$ is re-absorbed; notice that we have also re-absorbed the W coupling above)

$$Tr Mq\tilde{q} = M_{13}^X x_{32} x_{21} - M_{13}^Y y_{32} x_{21} + M_{43}^Y y_{32} x_{24} - M_{43}^X x_{32} x_{24}.
\tag{8}$$

The choice on the signs is such that they are the opposite of the terms in W containing the same mesons. Of course, they preserve the $SU(2)$ flavor symmetry of the theory.

¹We will henceforth call $X_{ij} \equiv X_{ij}^1, Y_{ij} \equiv X_{ij}^2, Z_{ij} \equiv X_{ij}^3$

The total superpotential for the dual theory thus reads

$$\begin{aligned}
W_{tot} = & X_{34}(Y_{41}X_{13} - M_{43}^Y) - Y_{34}(X_{41}X_{13} - M_{43}^X) + M_{13}^Y(Z_{34}X_{41} - y_{32}x_{21}) \\
& - M_{13}^X(Z_{34}Y_{41} - x_{32}x_{21}) + M_{43}^Y y_{32}x_{24} - M_{43}^X x_{32}x_{24}.
\end{aligned} \tag{9}$$

By using the F-term equations w.r.t. $X_{34}, Y_{34}, M_{43}^X, M_{43}^Y$ we see that these fields are all massive and can be integrated out. These F-term equations give

$$Y_{41}X_{13} = M_{43}^Y, \quad X_{41}X_{13} = M_{43}^X, \quad y_{32}x_{24} = X_{34}, \quad x_{32}x_{24} = Y_{34}, \tag{10}$$

which substituted in the above superpotential give

$$\begin{aligned}
W_{tot} = & M_{13}^Y(Z_{34}X_{41} - y_{32}x_{21}) - M_{13}^X(Z_{34}Y_{41} - x_{32}x_{21}) + \\
& Y_{41}X_{13}y_{32}x_{24} - X_{41}X_{13}x_{32}x_{24}.
\end{aligned} \tag{11}$$

This superpotential is exactly equivalent to the original one W , Eq.(1). In fact, the whole theory is equivalent to the original one. This is evident from the following redefinition of fields

$$\begin{aligned}
M_{13}^Y = \hat{Y}_{13}, \quad Y_{41} = \hat{Y}_{41}, \quad y_{32} = \hat{X}_{32}, \quad X_{13} = \hat{Z}_{13}, \quad Z_{34} = \hat{X}_{34}, \\
M_{13}^X = \hat{X}_{13}, \quad X_{41} = \hat{X}_{41}, \quad x_{32} = \hat{Y}_{32}, \quad x_{21} = \hat{X}_{21}, \quad x_{24} = \hat{X}_{24},
\end{aligned} \tag{12}$$

followed by the rotation of indexes

$$(1) \rightarrow (\hat{3}), \quad (2) \rightarrow (\hat{1}), \quad (3) \rightarrow (\hat{4}), \quad (4) \rightarrow (\hat{2}), \tag{13}$$

which give back simply the original theory.

In the non-conformal case, if we perform a Seiberg duality on node (2) there is no difference with respect to the calculations above, apart from the fact that now the gauge group on node (2) goes from $SU(N + 3M)$ to $SU(N - M)$. Again, the dual theory will be exactly as the original one but with the shift $N \rightarrow N - M$. Performing a second Seiberg duality on node ($\hat{2}$) we will find another equivalent theory with $N \rightarrow N - 2M$ with respect to the original one.² After k steps of the cascade $N \rightarrow N - kM$. For every step of the Seiberg cascade (before the ‘‘critical’’ one where some node has N_f equal or less than N_c), we can take W as in the original model, the only differences, step by step, being in the superpotential coupling λ . Hence we can treat each step of the cascade in a similar manner, as long as $N_f > N_c$ for every node. If $N = lM$, after $l - 1$ steps $N \rightarrow M$ and the field theory requires a more careful analysis.

²Remember that node ($\hat{2}$) is again the one whose coupling diverges before the other ones’, so the duality must be taken on this node.

3.2 The cascade at $N = M$ and beyond

Let us thus consider the cascade at $N = M$, where we end up with a $SU(M) \times SU(4M) \times SU(2M) \times SU(3M)$ theory. Since the gauge coupling for the second node generically blows up before the others, it makes sense to consider an energy scale where the other nodes are weakly coupled.

The $4M$ fundamental and anti-fundamental flavors of the $SU(4M)$ theory are given by ($A = 1, 2, \dots, 4M$ is a color index, a, k, i are flavor indexes)

$$Q = \left((X_{23})_a^A, (Y_{23})_a^A \right), \quad a = 1, \dots, 2M, \quad (14)$$

$$\tilde{Q} = \left((X_{12})_A^i, (X_{42})_A^k \right), \quad k = 1, 2, \dots, 3M, \quad i = 1, \dots, M, \quad (15)$$

arranged as $4M \times 4M$ matrices.

Let us now use gauge and flavor invariance (which is $SU(2) \times SU(2M) \times SU(3M) \times SU(M)$) so that, being painfully explicit with indexes, we get

$$\begin{aligned} Q &= \text{diag} \left((X_{23})_1^1, \dots, (X_{23})_{2M}^{2M}, (Y_{23})_1^{2M+1}, \dots, (Y_{23})_{2M}^{4M} \right), \\ \tilde{Q} &= \text{diag} \left((X_{12})_1^1, \dots, (X_{12})_M^M, (X_{42})_{M+1}^1, \dots, (X_{42})_{4M}^{3M} \right). \end{aligned} \quad (16)$$

The meson matrix $\mathcal{M} = Q \tilde{Q}$ will be given by

$$\begin{aligned} \mathcal{M} &= \text{diag} \left((X_{23})_1^1 (X_{12})_1^1, \dots, (X_{23})_M^M (X_{12})_M^M, (X_{23})_{M+1}^{M+1} (X_{42})_{M+1}^1, \dots, \right. \\ &\quad \left. (X_{23})_{2M}^{2M} (X_{42})_{2M}^M, (Y_{23})_1^{2M+1} (X_{42})_{2M+1}^{M+1}, \dots, (Y_{23})_{2M}^{4M} (X_{42})_{4M}^{3M} \right). \end{aligned} \quad (17)$$

We will use the following straightforward notations for the meson components³

$$M_{13}^X = X_{23} X_{12}, \quad M_{34}^X = X_{23} X_{42}, \quad M_{13}^Y = Y_{23} X_{12}, \quad M_{34}^Y = Y_{23} X_{42}. \quad (18)$$

The baryonic fields are formally given by

$$B \approx (X_{23})^{2M} (Y_{23})^{2M}, \quad \tilde{B} \approx (X_{12})^M (X_{42})^{3M}. \quad (19)$$

As it is well known, the theory without any other superpotential would have a moduli space described by $\det \mathcal{M} - B \tilde{B} = \Lambda^{2N_c}$. In fact the theory has also a superpotential

³Notice that we can formally write $\det \mathcal{M} = x^M y^{3M}$ after having identified M_{13}^X with a complex parameter x , and M_{34}^X, M_{34}^Y with y . The equation $\det \mathcal{M} = 0$ thus seems to be related (after recovering the explicit $SU(2)$ invariance which is broken by the above parameterization) to M copies of the complex cone over dP_1 which is in fact described by an equation of the form $x y^3 = z w^3$ [2].

term inherited by the original model on dP_1 , so that one has to consider the whole term

$$\begin{aligned}
W &= \text{Tr} [X_{34}Y_{41}X_{13} - Y_{34}X_{41}X_{13} - X_{34}M_{34}^Y + \\
&+ Y_{34}M_{34}^X + Z_{34}X_{41}M_{13}^Y - Z_{34}Y_{41}M_{13}^X] + \\
&+ \xi(\det \mathcal{M} - B\tilde{B} - \Lambda^{8M}),
\end{aligned} \tag{20}$$

where ξ is a Lagrange multiplier.

Let us now fix gauge and global invariance on the other nodes of the quiver, i.e. solve the other three D-term equations.

- Node (3) has $N_f = 9M$ and $N_c = 2M$. The fundamental flavors can be represented as a (symbolic) row (X_{34}, Y_{34}, Z_{34}) . Each block behaves as $n_f = 3M$ flavors. Fixing gauge and global symmetries only $2M$ components of each block survive. Let us consider as an example, the theory with $M = 1$. We choose as non vanishing elements $(X_{34})_2^1, (X_{34})_3^2, (Y_{34})_1^2, (Y_{34})_2^1, (Z_{34})_2^2, (Z_{34})_3^1$. The anti-fundamental flavors can be represented as a column (X_{13}, X_{23}, Y_{23}) . The first block behaves as $n_f = M$ flavors while the other two as $n_f = 4M$ each. In the $M = 1$ case, we can take as non vanishing components $(X_{13})_1^1, (X_{23})_1^1, (X_{23})_2^2, (Y_{23})_1^3, (Y_{23})_2^4$, the latter being consistent with the choices on node (2).
- Node (4) has $N_f = 6M$, $N_c = 3M$. The fundamental flavors can be represented by the row (X_{42}, X_{41}, Y_{41}) , the first block behaving as $n_f = 4M$ flavors, the other two as $n_f = M$. In the case $M = 1$ we can take $(X_{42})_2^1, (X_{42})_3^2, (X_{42})_4^3, (X_{41})_1^1, (Y_{41})_1^3$. The anti-fundamental flavors are a column (X_{34}, Y_{34}, Z_{34}) and a consistent gauge fixing save the same components as appearing for node (3) above.
- Finally node (1) has $N_f = 6M$ and $N_c = M$. A consistent gauge fixing on the fundamentals X_{12}, X_{13} and the anti-fundamentals X_{41}, Y_{41} selects the same components as the ones just selected before.

We are now ready to examine the F-term equations induced from the superpotential (20). Let us first consider those related to ξ, B, \tilde{B} .

One class of solutions of these three equations is $B = \tilde{B} = 0, \det \mathcal{M} = \Lambda^{8M}$, i.e. the mesonic branch. Let us show that this class is in fact empty. The F-term equations w.r.t. Y_{34} and X_{34} give

$$M_{34}^X = X_{41}X_{13}, \quad M_{34}^Y = Y_{41}X_{13}. \tag{21}$$

From the parameterizations introduced previously, the components of M_{34}^X saved by the diagonalization are put to zero by the above equations. Thus $\det \mathcal{M} = 0$, in contradiction with $\det \mathcal{M} = \Lambda^{8M}$.

The only other class of possible solutions is the one with $\xi = \det \mathcal{M} = 0$, $B\tilde{B} = -\Lambda^{8M}$, i.e. the baryonic branch. As in the calculation in section 3.1, the massive mesons M_{34}^X, M_{34}^Y can be integrated out by means of (21). The theory flows in the IR, under the scale Λ of the node (2), towards a theory where node (4) is at strong coupling. The superpotential is

$$W = \text{Tr} [N_{31}^X M_{13}^Y - N_{31}^Y M_{13}^X] + W_{ADS}, \quad (22)$$

where the Affleck-Dine-Seiberg superpotential W_{ADS} involving the mesonic fields $\mathcal{N} = (N_{31}^X, N_{31}^Y) \equiv (Z_{34}X_{41}, Z_{34}Y_{41})$ reads

$$W_{ADS} = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det \mathcal{N}} \right)^{\frac{1}{N_c - N_f}} = M \left(\frac{\Lambda^{7M}}{\det \mathcal{N}} \right)^{\frac{1}{M}}. \quad (23)$$

It is non-perturbatively generated because of the fact that node (4) has now $N_f (= 2M)$ less than $N_c (= 3M)$. The IR quiver triangle is reported in figure 2.

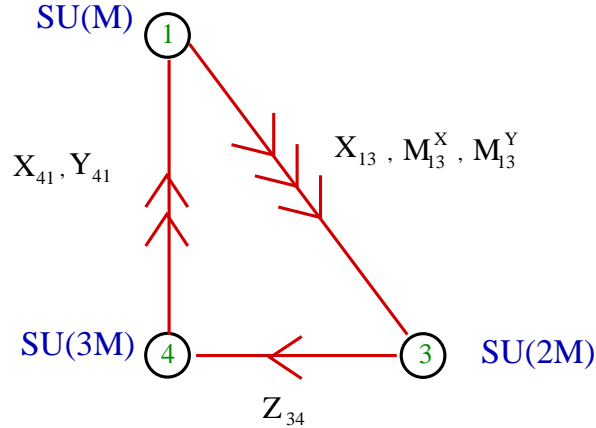


Figure 2: The quiver triangle at the bottom of the cascade.

It is now easy to check that the superpotential (22) admits no supersymmetric vacuum. Indeed, the F-term equations for the M_{13} fields imply $N_{31} = 0$, which are not consistent solutions for the remaining equations. Hence, as anticipated, supersymmetry is dynamically broken in this model. This also implies that the dual geometric deformation that should capture the IR dynamics of the non-conformal $Y^{2,1}$ cascade should be

non-supersymmetric. This agrees with the statement in [6] that there are no complex deformations for the complex cone over the first del Pezzo surface.

Our analysis is quite generic and although the corresponding explicit computations might be more and more cumbersome, we believe the same conclusion to apply to the full class of $Y^{p,p-1}$ and $Y^{p,1}$ manifolds, of which the case we have considered can be seen as a master example. In fact, in [6] it was shown that a supersymmetric complex deformation of the cone over $Y^{p,q}$ for any possible value of q cannot occur. This suggests that the full $Y^{p,q}$ series might undergo dynamical supersymmetry breaking, although the IR dynamics for generic q has not been fully understood, yet.

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