Fixing Moduli in Exact Type IIA Flux Vacua

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Type IIA flux compactifications with O6-planes have been argued from a four dimensional effective theory point of view to admit stable, moduli free solutions. We discuss in detail the ten dimensional description of such vacua and present exact supersymmetric solutions in the case when the O6-charge is smoothly distributed: the geometry is Calabi-Yau and the dilaton is constant. In the localized case, the solution is a half-flat, non-Calabi-Yau metric but still with constant dilaton. Finally, using the ten dimensional description we show how all moduli are stabilized and reproduce precisely the results of de Wolfe et *al.* [1]. Based on [7].

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1   Introduction

String vacua with magnetic fields turned on in the compact extra dimensions ("flux compactifications") have attracted much attention since many years. The crucial feature is that their contribution to the energy of the system depends on the moduli of the geometry which supports them, so to provide an effective four dimensional potential for the moduli fields, possibly lifting some or all of them. Moreover the fluxes are quantized, so their strength is not another continuously varying free parameter. Together these give a mechanism for moduli stabilization.

From this point of view, particularly interesting are Type IIA vacua, since in many models a complete stabilization of the moduli can be accomplished at tree level in supergravity, that is without appealing to α' corrections or actually to string loops or non-perturbative effects. This is because the classical effective moduli potential generated by the tree level IIA supergravity action (supplemented with O6-plane sources) has stable *isolated* minima [1, 2, 3].

Being concrete [1, 3], if we consider Type IIA theory on a Calabi-Yau three-fold, switching on R-R fluxes F_0, F_2, G_4 stabilizes all the Kähler moduli of the internal manifold. To stabilize also the complex structure moduli, one has to introduce a NS-NS 3-form flux H_3. However this gives a tadpole for the D6-charge, that can be cancelled by the introduction of orientifold six-planes. They also help in halving the complex structure moduli. The full system then stabilizes all the moduli, essentially at leading order in α' and g_s.

An approximation widely used in the literature to study string vacua in the supergravity regime, in which R-R and NS-NS field-strengths are switched on, is the so called "Calabi-Yau with fluxes" approximation. It resides in taking the flux contribution to the supergravity action and the equations of motion small compared to the curvature contribution, and it works in the large volume limit. Being the fluxes quantized by Dirac quantization condition, the minimum amount of a p-form field-strength is $F_p \sim (\alpha')^{(p-1)/2}$. Then one requires the contribution of fluxes to the action to be small compared to the Einstein term R, which is of order L^{-2} (with L the characteristic length of the manifold). This gives $\alpha'/L^2 \ll 1$. In other words, we must be in the limit of large compactification manifold with respect to the string length,

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which anyway is the regime of applicability of supergravity. Under these conditions, one can neglect the backreaction of fluxes on the geometry, and work with Calabi-Yau metric solutions. Of course one has to be careful to remember that in the action the various contributions are weighted by the string coupling, and both the dilaton and the volume are (possibly) determined by the fluxes themselves, so it is not always possible to keep the fluxes to their minimal amount while increasing the volume.

Such an approximation can also hide subtle phenomena. For example, even the minimum amount of flux allowed by Dirac quantization condition can, if correctly taken into account, change the topology of the manifold as well as cohomology classes and so on. It is not clear how the Calabi-Yau geometry can approximate disconnected topologies.

Another powerfull tool in the discussion of moduli stabilization is the reduction to a four dimensional effective theory, with the computation of the 4d effective potential for light fields. Then vacua are found by looking at its stationary points. Many issues are studied more easily than with a ten dimensional approach. For instance, stability is checked just by looking at the second derivatives of the potential. Moreover non-supersymmetric vacua can be analyzed as well, and there is some control on exotic vacua, such as non-geometric vacua [4].

The approach has anyway also bad facets. First of all, one has to work in a Kaluza-Klein approximation, expanding the ten dimensional fields on a Kaluza-Klein basis (suitable for a Calabi-Yau geometry without fluxes) and then keeping only light fields. In general there is no guaranty that the same basis furnishes the light fields in presence of background fluxes.[1] Moreover, even finding stable minima of the 4d effective potential, one cannot be in general sure about the existence of a 10d lift. All these reasons brought us to undertake the ten dimensional approach, and the following discussion is based on [7].

In [1] Type IIA string vacua in the supergravity limit were constructed, by compactifying on the orbifold $T^6/(\mathbb{Z}_3)^2$ and then adding O6-planes. Such vacua were analyzed classically and in the large volume regime, by computing the 4d effective potential and deriving it from a superpotential. Our main result will be to provide a full 10d description of their vacua with backreacting fluxes [7], reproducing their results.

A key ingredient is a smearing procedure. Indeed we show that in no case with non-trivial fluxes and localized sources (O6-planes) the geometry is Calabi-Yau, and this makes it hard to exhibit explicit examples. We overcome the problem by homogeneously distributing the O6-charge on the internal manifold, which makes it possible to find Calabi-Yau solutions with backreacting non-trivial fluxes. We stress that all these solutions have *constant* dilaton, and the string coupling can be kept arbitrarily small everywhere.

Another approach attempted to give a full description of the vacua in [1] appeared in [8]. The authors performed a double T-duality of the system to get rid of the F_0 and H_3 fluxes, and then uplifted to M-theory.

2   Type IIA Supergravity Solutions with O6-planes

The starting point to give a full ten dimensional description in the supergravity regime of the vacua constructed in [1] are the Lust-Tsimpis solutions [9] (see also [10]). They are the most general solutions of Type IIA supergravity compactified on a warped product of AdS_4 and an internal $SU(3)$-structure manifold, which preserve $\mathcal{N} = 1$ supersymmetry in 4d. All the fluxes compatible with the Poincaré symmetry of AdS_4 are switched on. These include also a "mass parameter" $F_0 \equiv m$, which is one of the R-R fluxes provided by IIA string theory and at low energy is described within the so called massive Type IIA supergravity [11]. Then we will add O6-planes to them in a way to preserve $\mathcal{N} = 1$ supersymmetry, in order to be able to realize, among the others, the $T^6/(\mathbb{Z}_3)^2$ orientifold model of [1].

The internal manifold is taken to be an $SU(3)$-structure manifold, so to have one globally defined spinor that, combined with the four dimensional ones, lets us construct four 10d Killing spinors. An $SU(3)$-structure manifold is described by a Kähler 2-form J and an holomorphic 3-form Ω, like on a Calabi-Yau 3-fold. But, unlike that case, they are not closed. Rather their differentials, conveniently split

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[1] For some attempts to do the Kaluza-Klein reduction with fluxes see [5, 6].

according to $SU(3)$ representations, define the torsion classes and measure the deviation from the Calabi-Yau geometry [12]:

$$dJ = -\frac{3}{2}\,\mathbb{I}\mathrm{m}\left(\mathcal{W}_1^{1\oplus 1}\Omega^*\right) + \mathcal{W}_4^{3\oplus\bar{3}} \wedge J + \mathcal{W}_3^{6\oplus\bar{6}} \tag{1}$$

$$d\Omega = \mathcal{W}_1^{1\oplus 1} J \wedge J + \mathcal{W}_2^{8\oplus 8} \wedge J + \mathcal{W}_5^{3\oplus\bar{3}} \wedge \Omega \,. \tag{2}$$

Here $\mathcal{W}_i$ are the five torsion classes, and the specific $SU(3)$ representation is indicated.

In the present setup, supersymmetry only allows the non-vanishing torsion classes $\mathcal{W}_1^-$ and $\mathcal{W}_2^-$ [9], where minus stands for the imaginary part. This restricts the possible geometries to a special subclass of *half-flat* manifolds.

We can introduce orientifold six-planes in the picture. These are not genuine supergravity objects, rather they are defined by projecting the Type IIA string theory. However the supergravity action can be enriched with terms that describe the interaction of O-planes with the low energy fields. In Type IIA string theory, an O6-plane is obtained by modding out the theory by the discrete symmetry operator $\mathcal{O} = \Omega_p(-1)^{F_L}\sigma^*$, where Ω_p is the world-sheet parity, F_L is the left-moving spacetime fermion number and σ is an isometrical involution of the background. The fixed point locus of σ is the O6-plane. In Type IIA string theory an O6-plane is a BPS object that preserves half of the supersymmetries, by relating the two Majorana-Weyl supersymmetry parameters according to $\epsilon_\pm = \mathcal{O}\epsilon_\mp$. In the vacua we are going to construct we want to preserve 4d Poincaré symmetry, so the O6-planes will span the AdS_4 factor and will wrap a three dimensional cycle in the internal manifold. In order to obtain again vacua with four supercharges, we must place the O6-planes in such a way to preserve the four supercharges of the background. This is achieved by wrapping the planes on supersymmetric 3-cycles Σ, which in the case of $SU(3)$-structure manifolds are generalized special Lagrangian 3-cycles[2] [13]:

$$J|_\Sigma = 0 \qquad\qquad \mathbb{R}\mathrm{e}\,\Omega|_\Sigma = 0 \qquad\qquad \mathbb{I}\mathrm{m}\,\Omega|_\Sigma = -\mathrm{Vol}_\Sigma \,. \tag{3}$$

The description of the low energy dynamics is achieved, at leading order in α', by adding to the Type IIA supergravity action a Dirac-Born-Infeld and a Wess-Zumino term for the O6-planes. The main modification is a violation of the Bianchi identity for the R-R 2-form field-strength F_2, since O6-planes couple to the R-R 7-form potential C_7 and are thus magnetic sources for F_2:

$$dF_2 = 2mH_3 - 2\mu_6\,\delta_3 \,, \tag{4}$$

where δ_3 is the O6-charge distribution, a 3-form localized on Σ. H_3 is the NS-NS 3-form field-strength and μ_6 is the Einstein frame D6-charge. By imposing supersymmetry we can find the following solutions, where everything is determined by the geometry:

$$F_2 = \frac{f}{9}e^{-\phi/2}J + \tilde{F}_2 \qquad\qquad H_3 = \frac{4m}{5}e^{7\phi/4}\,\mathbb{R}\mathrm{e}\,\Omega \qquad\qquad -\frac{4i}{9}f\,e^{\phi/4} = \mathcal{W}_1^-$$

$$G_4 = f\,dVol_4 + \frac{3m}{5}e^\phi J \wedge J \qquad \phi\,, f\,, \Delta = \mathrm{constant} \qquad -i\,e^{3\phi/4}\tilde{F}_2 = \mathcal{W}_2^- \,. \tag{5}$$

In particular, the geometry is again a special subclass of half-flat. Some explanations are in order. G_4 is the R-R 4-form field-strength, f is a Freund-Rubin parameter, ϕ is the IIA dilaton and Δ is the warp factor which is found to be constant. $\tilde{F}_2$ is the irreducible component of F_2 transforming in the **8** of $SU(3)$, and is constrained by the O6-charge distribution:

$$d\tilde{F}_2 = -\frac{2}{27}e^{-\phi/4}\left(f^2 - \frac{108}{5}m^2 e^{2\phi}\right)\mathbb{R}\mathrm{e}\,\Omega - 2\mu_6\,\delta_3 \,. \tag{6}$$

Generalizing the argument in [9] one can show that these are actually solutions of the equations of motion.

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[2] Supersymmetry forces σ to be antiholomorphic with respect to the almost complex structure J, and the pull-back of the NS-NS 2-form potential B_2 on Σ to vanish.

From (5) and (6), keeping δ_3 a localized form, we can derive the inequality $f^2 \geq \frac{108}{5} m^2 e^{2\phi}$, from which an important observation follows. As long as the mass parameter m is different from zero, $\mathcal{W}_1^-$ is non-trivial and there are *no Calabi-Yau* solutions (by careful analysis, $m = 0$ leads to trivial Calabi-Yau solutions without fluxes and O6-planes). Constructing explicit examples of half flat manifolds which solve the equations, even without O6-planes, is difficult (for some attempts see [6, 9]). One possible way out is to apply a smearing procedure, much like the smearing of D-branes: the O6-charge is homogeneously distributed over the internal manifold, while the orientifold projection is kept untouched.[3] By substituting the localized charge distribution δ_3 with the smeared one:

$$\mu_6 \, \delta_3^{\text{smeared}} = \frac{4m^2}{5} \, e^{7\phi/4} \, \mathbb{R}\text{e} \, \Omega \tag{7}$$

it is possible to find solutions with $m^2 > 0$ and $f = 0$, $F_2 = \tilde{F}_2 = 0$. These are Calabi-Yau geometries with non-trivial backreacting fluxes.

More generally, every Calabi-Yau 3-fold that admits an anti-holomorphic involution provides a solution of our equations, with O6-planes along the fixed point locus of the involution. Moreover, as we will see in the next section, in such geometries all moduli can be stabilized.

By integrating (7) on a cycle Γ symplectic partner of Σ, it is possible to show that the dilaton is fixed to

$$e^{7\phi/4} = \frac{5\mu_6}{8m^2 \sqrt{\text{Vol}_6}} \, , \tag{8}$$

and the 4d cosmological constant is fixed as well. Summarizing, the solution is completely described by an internal Calabi-Yau manifold defined by J and Ω, with an antiholomorphic isometrical involution σ. Further constraints come from the integral quantization of fluxes, and this mechanism provides the stabilization of *all* moduli.

3   Moduli Stabilization

The next step is to use the class of solutions previously found to discuss the possible presence of free moduli. The question is – is there a continuously connected family of vacua, or do they form only a discrete set? We will answer by exploiting the class of smeared Calabi-Yau solutions, but the same discussion could be carried over in the localized case.

First of all we split the VEV of a field strength in two pieces, *e.g.* $H_3 = H_3^{\text{flux}} + dB_2$. The first piece, closed and non-trivial in cohomology, represents the integral flux contribution, while the second one, exact, encodes any other oscillation or deformation. In such a separation there is of course some arbitrariness. From our smeared Calabi-Yau solution, we know H_3 to be harmonic, so the natural choice is to take H_3^{flux} harmonic as a representative of the cohomology class, and then use the gauge freedom to put also B_2 harmonic. The fact that this is always possible shows that cannot be any physical non-harmonic component in B_2: these have vanishing VEV and are massive. Then we can expand H_3^{flux} and B_2 in an harmonic basis and substitute in the solution (5), to see if free parameters are left.

We can easily handle the $T^6/(\mathbb{Z}_3)^2$ orientifold model of [1]. The geometry is the orbifold limit of a Calabi-Yau manifold with $h^{2,1} = 0$, $h^{1,1} = 12$, where 9 of the 12 Kähler moduli arise from the blow-up modes of 9 $\mathbb{Z}_3$ singularities. We will not discuss the twisted moduli here, since they fit well in the general Calabi-Yau case [7]. It's useful to introduce an integral basis of harmonic forms: the 2-forms (odd under σ) w_i such that $\int w_1 \wedge w_2 \wedge w_3 = 1$ and $w_i^2 = 0$; the 4-forms (even under σ) $\tilde{w}^i = w_j \wedge w_k$, where j, k

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[3] We must note that while the smearing of a large number of branes is perfectly justified, the smearing of a small number is less, as well as the fact that the orientifold projection cannot be smeared.

are the two values among $1, 2, 3$ besides i. The field-strengths are split according to:

$$F_2 = F_2^{\text{flux}} + dA_1 + 2m\, B_2$$

$$H_3 = H_3^{\text{flux}} + dB_2$$

$$G_4 = G_4^{\text{flux}} + f\, dVol_4 + dC_3 + B_2 \wedge dA_1 + mB_2^2$$

$$e^{\phi/2} * G_4 = F_6^{\text{flux}} + dC_5 - H_3 \wedge C_3 + B_2 \wedge G_4^{\text{flux}} + \tfrac{1}{3}m\, B_2^3 \,. \tag{9}$$

Notice that being A_1 harmonic, it is actually vanishing on a Calabi-Yau manifold. Then we expand on the harmonic basis both the fluxes, integrally quantized, and the forms possibly containing free moduli:

$$F_2^{\text{flux}} = f^i\, w_i \qquad\qquad\qquad B_2 = b^i\, w_i \tag{10}$$

$$G_4^{\text{flux}} = e_i\, \tilde{w}^i \qquad\qquad\qquad J = e^{-\phi/2}\, v^i\, w_i \tag{11}$$

$$H_3^{\text{flux}} = \frac{p}{4\sqrt{\text{Vol}_6}}\, \mathbb{Re}\,\Omega \qquad\qquad C_3 = \frac{\xi}{4\sqrt{\text{Vol}_6}}\, \mathbb{Im}\,\Omega \tag{12}$$

$$F_6^{\text{flux}} = e_0\, w_1 \wedge w_2 \wedge w_3 \,, \tag{13}$$

where f^i, e_i, p and e_0 (as well as m) are quantized in suitable units, and $\text{Vol}_6 = e^{-3\phi/2}\, v^1 v^2 v^3$. The could-be moduli are v^i, ϕ and the axions b^i and ξ, which pair into 4d chiral fields. The only non-trivial tadpole cancellation condition is $m\, p = \mu_6$. Substituting in the smeared solution we get $b^i = -f^i/2m$. Taking for simplicity $f^i = 0$ we also get:

$$v^i = \frac{1}{|e_i|}\sqrt{\frac{5}{6}\frac{e_1 e_2 e_3}{m}} \qquad e^\phi = \frac{3}{4}\mu_6 \left(\frac{5}{6}\frac{1}{m^5\, e_1 e_2 e_3}\right)^{1/4} \qquad p\,\xi = e_0 \,. \tag{14}$$

We could also compute the 4d cosmological constant Λ [7]. Everything is in perfect agreement with the results of [1].

The same machinery applies to general Calabi-Yau solutions with general backreacting fluxes and smeared O6-planes, as discussed in [7]. In general there are enough equations to fix almost all the would-be moduli: all the Kähler and complex structure moduli, the dilaton, all axions from B_2 and one axion from C_3. The remaining C_3 axions turn out not to be fixed yet, but this is not a phenomenological problem. Taking values on a compact S^1, there cannot be any dangerous runaway behavior and any other contribution to their effective potential lifts them.

References

[1] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, JHEP **0507**, 066 (2005) [arXiv:hep-th/0505160].

[2] T. W. Grimm and J. Louis, Nucl. Phys. B **718**, 153 (2005) [arXiv:hep-th/0412277].
P. G. Camara, A. Font and L. E. Ibanez, JHEP **0509**, 013 (2005) [arXiv:hep-th/0506066].

[3] G. Villadoro and F. Zwirner, JHEP **0506**, 047 (2005) [arXiv:hep-th/0503169].

[4] S. Kachru, M. B. Schulz, P. K. Tripathy and S. P. Trivedi, JHEP **0303**, 061 (2003) [arXiv:hep-th/0211182].
S. Hellerman, J. McGreevy and B. Williams, JHEP **0401**, 024 (2004) [arXiv:hep-th/0208174].
A. Flournoy, B. Wecht and B. Williams, Nucl. Phys. B **706**, 127 (2005) [arXiv:hep-th/0404217].
J. Shelton, W. Taylor and B. Wecht, JHEP **0510**, 085 (2005) [arXiv:hep-th/0508133].
A. Lawrence, M. B. Schulz and B. Wecht, JHEP **0607**, 038 (2006) [arXiv:hep-th/0602025].

[5] S. Gurrieri, J. Louis, A. Micu and D. Waldram, Nucl. Phys. B **654**, 61 (2003) [arXiv:hep-th/0211102].
S. Gurrieri and A. Micu, Class. Quant. Grav. **20**, 2181 (2003) [arXiv:hep-th/0212278].
S. Fidanza, R. Minasian and A. Tomasiello, Commun. Math. Phys. **254**, 401 (2005) [arXiv:hep-th/0311122].
A. Tomasiello, JHEP **0506**, 067 (2005) [arXiv:hep-th/0502148].
M. Grana, J. Louis and D. Waldram, JHEP **0601**, 008 (2006) [arXiv:hep-th/0505264].
A. K. Kashani-Poor and R. Minasian, arXiv:hep-th/0611106.

[6] T. House and E. Palti, Phys. Rev. D **72**, 026004 (2005) [arXiv:hep-th/0505177].

[7] B. S. Acharya, F. Benini and R. Valandro, JHEP **0702**, 018 (2007) [arXiv:hep-th/0607223].

[8] T. Banks and K. van den Broek, arXiv:hep-th/0611185.

[9] D. Lust and D. Tsimpis, JHEP **0502**, 027 (2005) [arXiv:hep-th/0412250].

[10] K. Behrndt and M. Cvetic, Phys. Rev. Lett. **95**, 021601 (2005) [arXiv:hep-th/0403049].
     K. Behrndt and M. Cvetic, Nucl. Phys. B **708**, 45 (2005) [arXiv:hep-th/0407263].

[11] L. J. Romans, Phys. Lett. B **169**, 374 (1986).

[12] S. Chiossi and S. Salamon, arXiv:math.dg/0202282.

[13] J. F. G. Cascales and A. M. Uranga, JHEP **0411**, 083 (2004) [arXiv:hep-th/0407132].