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Fundamental strings in SFT

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Abstract

In this letter we show that vacuum string field theory contains exact solutions that can be interpreted as macroscopic fundamental strings. They are formed by a condensate of infinitely many completely space–localized solutions (D0-branes).

1 Introduction

Vacuum string field theory (VSFT) is a version of Witten's open string field theory which is conjectured to represent string theory at the tachyon condensation vacuum [1]. Its action is formally the same as the original Witten theory except that the BRST charge takes a simplified form: it has been argued that it can be expressed simply in terms of the ghost creation and annihilation operators. By virtue of this simplification it has been possible to determine exact classical solutions which have been shown to represent D-branes. The existence of such solutions confirms the conjecture at the basis of VSFT. The tachyon condensation vacuum physics can only represent closed string theory and thus, if VSFT is to represent string theory at the tachyon condensation vacuum, it should be able to describe closed string theory in the sense of [2]. The above D-brane solutions, expressed in the open string language of VSFT, correspond precisely to objects that in closed string language appear either as boundary state or as solutions of low energy effective actions.

Recently it has been possible to find in VSFT an exact time-dependent solution, [3], with the characteristics of a rolling tachyon [4]. A rolling tachyon describes in various languages (effective field theory, BCFT, SFT) the decay of unstable D-branes. It is by now clear that the final product of a brane decay is formed by massive closed string states. However it has been shown that, in the presence of a background electric field also (macroscopic) fundamental strings appear as final products of a brane decay. Now, since our aim is to describe a brane decay in the framework of VSFT we must show first of all that such

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fundamental strings exist as solutions of VSFT. In this letter we want to present some evidence that such solutions do exist.

The letter is organized as follows. In the next section we collect some well–known formulas which are needed in the sequel. In section 3 we show in a rather informal way how to construct new one-dimensional solutions as condensate of D0–branes. In section 4 we give a more motivated account of the same construction by introducing a background B field. In the last section we provide evidence that the new solutions represent fundamental strings.

2 A reminder

In this section we recall the notation and some useful formulas. The VSFT action is

$$\mathcal{S}(\Psi) = -\left(\frac{1}{2}\langle\Psi|\mathcal{Q}|\Psi\rangle + \frac{1}{3}\langle\Psi|\Psi * \Psi\rangle\right) \tag{1}$$

where

$$\mathcal{Q} = c_0 + \sum_{n>0} (-1)^n \left(c_{2n} + c_{-2n}\right) \tag{2}$$

the ansatz for nonperturbative solutions is in the factorized form

$$\Psi = \Psi_m \otimes \Psi_g \tag{3}$$

where Ψ_g and Ψ_m depend purely on ghost and matter degrees of freedom, respectively. The equation of motion splits into

$$\mathcal{Q}\Psi_g = -\Psi_g *_g \Psi_g \tag{4}$$

$$\Psi_m = \Psi_m *_m \Psi_m \tag{5}$$

where $*_g$ and $*_m$ refer to the star product involving only the ghost and matter part. The action for this type of solution is

$$\mathcal{S}(\Psi) = -\frac{1}{6}\langle\Psi_g|\mathcal{Q}|\Psi_g\rangle\langle\Psi_m|\Psi_m\rangle \tag{6}$$

$\langle\Psi_m|\Psi_m\rangle$ is the ordinary inner product. We will concentrate on the matter part, eq.(5), assuming the existence of a universal ghost solution. The solutions are projectors of the $*_m$ algebra. The $*_m$ product is defined as follows

$$_{123}\langle V_3|\Psi_1\rangle_1|\Psi_2\rangle_2 =_3 \langle\Psi_1 *_m \Psi_2|, \tag{7}$$

see [5, 6, 7, 8] for the definition of the three string vertex $_{123}\langle V_3|$. In the following we need both translationally invariant and non-translationally invariant solutions. Although there is a great variety of such solutions we will stick to those introduced in [9], i.e. the sliver and the lump. The former is translationally invariant and is defined by

$$|\Xi\rangle = \mathcal{N} e^{-\frac{1}{2}a^\dagger\cdot S\cdot a^\dagger}|0\rangle, \qquad a^\dagger\cdot S\cdot a^\dagger = \sum_{n,m=1}^{\infty} a_n^{\mu\dagger} S_{nm} a_m^{\nu\dagger}\eta_{\mu\nu} \tag{8}$$

where $S = CT$ and

$$T = \frac{1}{2X}(1 + X - \sqrt{(1+3X)(1-X)}) \tag{9}$$

with $X = CV^{11}$. In proving that this is a solution it is crucial that the matrices $X = CV^{11}, X_+ = CV^{12}$ and $X_- = CV^{21}$, are mutually commuting and commute with $T = CS = SC$, where $C_{nm} = (-1)^n \delta_{nm}$. The normalization constant $\mathcal{N}$ needs being regularized and is formally vanishing. It has been shown in other papers how this problem could be dealt with, [10, 11].

The lump solution is engineered to represent a lower dimensional brane (Dk-branes), therefore it will have $(25-k)$ transverse space directions along which translational invariance is broken. Accordingly we split the three string vertex into the tensor product of the perpendicular part and the parallel part

$$|V_3\rangle = |V_{3,\perp}\rangle \otimes |V_{3,\parallel}\rangle \tag{10}$$

The parallel part is the same as in the sliver case while the perpendicular part is modified as follows. Following [9], we denote by x^α, p^α, $\alpha = 1, ..., k$ the coordinates and momenta in the transverse directions and introduce the canonical zero modes oscillators

$$a_0^{(r)\alpha} = \frac{1}{2}\sqrt{b}\hat{p}^{(r)\alpha} - i\frac{1}{\sqrt{b}}\hat{x}^{(r)\alpha}, \qquad a_0^{(r)\alpha\dagger} = \frac{1}{2}\sqrt{b}\hat{p}^{(r)\alpha} + i\frac{1}{\sqrt{b}}\hat{x}^{(r)\alpha}, \tag{11}$$

where b is a free parameter. Denoting by $|\Omega_b\rangle$ the oscillator vacuum ($a_0^\alpha|\Omega_b\rangle = 0$), in this new basis the three string vertex is given by

$$|V_{3,\perp}\rangle' = K\, e^{-E'}|\Omega_b\rangle \tag{12}$$

with

$$K = \left(\frac{\sqrt{2\pi b^3}}{3(V_{00} + b/2)^2}\right)^{\frac{k}{2}}, \qquad E' = \frac{1}{2}\sum_{r,s=1}^{3}\sum_{M,N\geq 0} a_M^{(r)\alpha\dagger} V_{MN}^{\prime rs} a_N^{(s)\beta\dagger}\eta_{\alpha\beta} \tag{13}$$

where M, N denote the couple of indices $\{0, m\}$ and $\{0, n\}$, respectively. The coefficients $V_{MN}^{\prime rs}$ are given in Appendix B of [9]. The new Neumann coefficients matrices $V^{\prime rs}$ satisfy the same relations as the V^{rs} ones. In particular one can introduce the matrices $X^{\prime rs} = CV^{\prime rs}$, where $C_{NM} = (-1)^N \delta_{NM}$, which turn out to commute with one another. The lump solution $|\Xi_k'\rangle$ has the form (8) with S along the parallel directions and S replaced by S' along the perpendicular ones. In turn $S' = CT'$ and T' has the same form as T eq.(9) with X replaced by X'. The normalization constant $\mathcal{N}'$ is defined in a way analogous to $\mathcal{N}$ and the same remarks hold for it. It can be verified that the ratio of tensions for such solutions is the appropriate one for Dk–branes. Moreover the space profile of these solutions in the transverse direction is given by a Gaussian (see [12, 14]). This reinforces the interpretation of these solutions as branes.

3 Constructing new solutions

In this section we would like to show how qualitatively new solutions to (5) can be constructed by accretion of infinite many lumps. Let us start from a lump solution representing a D0–brane as introduced in the previous section: it has a Gaussian profile in all space directions, the form of the string field – let us denote it $|\Xi_0'\rangle$ – will be the same as (9) with S replaced by S', while the $*$–product will be determined by the primed three strings vertex (12). Let us pick one particular space direction, say the α–th. For simplicity in the following we will drop the corresponding label from the coordinate $\hat{x}^\alpha$, momenta $\hat{p}^\alpha$ and oscillators a^α along this direction. Next we need the same solution displaced by an amount s in the positive x direction (x being the eigenvalue of $\hat{x}$). The appropriate solution has been constructed by Rastelli, Sen and Zwiebach, [15]:

$$|\Xi_0'(s)\rangle = e^{-is\hat{p}}|\Xi_0'\rangle \tag{14}$$

It satisfies $|\Xi_0'(s)\rangle * |\Xi_0'(s)\rangle = |\Xi_0'(s)\rangle$. Eq.(14) can be written explicitly as

$$|\Xi_0'(s)\rangle = \mathcal{N}' e^{-\frac{s^2}{2b}(1-S'_{00})} \exp\left(-\frac{is}{\sqrt{b}}((1-S')\cdot a^\dagger)_0\right) \exp\left(-\frac{1}{2}a^\dagger \cdot S' \cdot a^\dagger\right)|\Omega_b\rangle \tag{15}$$

where $((1-S'_{00})\cdot a^\dagger)_0 = \sum_{N=0}^{\infty}((1-S')_{0N}a_N^\dagger)$ and $a^\dagger \cdot S' \cdot a^\dagger = \sum_{N,M=0}^{\infty} a_N^\dagger S'_{NM} a_M^\dagger$; $\mathcal{N}'$ is the $|\Xi_0'\rangle$ normalization constant. Moreover one can show that

$$\langle \Xi_0'(s)|\Xi_0'(s)\rangle = \langle \Xi_0'|\Xi_0'\rangle \tag{16}$$

The meaning of this solution is better understood if we make its space profile explicit by contracting it with the coordinate eigenfunction

$$|\hat{x}\rangle = \left(\frac{2}{\pi b}\right)^{\frac{1}{4}} \exp\left(-\frac{x^2}{b} - i\frac{2}{\sqrt{b}}a_0^\dagger x + \frac{1}{2}a_0^\dagger a_0^\dagger\right)|\Omega_b\rangle \tag{17}$$

The result is

$$\langle \hat{x}|\Xi_0'(s)\rangle = \left(\frac{2}{\pi b}\right)^{\frac{1}{4}} \frac{\mathcal{N}'}{\sqrt{1+S'_{00}}} e^{-\frac{1-S'_{00}}{1+S'_{00}}\frac{(x-s)^2}{b}} e^{-\frac{2i}{\sqrt{b}}\frac{x-s}{1+S'_{00}}S'_{0m}a_m^\dagger} e^{-\frac{1}{2}a_n^\dagger W_{nm} a_m^\dagger}|0\rangle \tag{18}$$

where $W_{nm} = S'_{nm} - \frac{S'_{n0}S'_{0m}}{1+S'_{00}}$. It is clear that (18) represents the same Gaussian profile as $|\Xi_0'\rangle = |\Xi_0'(0)\rangle$ shifted away from the origin by s.

It is important to remark now that two such states $|\Xi_0'(s)\rangle$ and $|\Xi_0'(s')\rangle$ are $*$–orthogonal and *bpz*–orthogonal provided that $s \neq s'$. For we have

$$|\Xi_0'(s)\rangle * |\Xi_0'(s')\rangle = e^{-\mathcal{C}(s,s')}|\Xi_0'(s,s')\rangle \tag{19}$$

where the state $|\Xi_0'(s,s')\rangle$ becomes proportional to $|\Xi_0'(s)\rangle$ when $s = s'$ and needs not be explicitly written down otherwise; while

$$\mathcal{C}(s,s') = -\frac{1}{2b}\left[(s^2+s'^2)\left(\frac{T'(1-T')}{1+T'}\right)_{00} + ss'\left(\frac{(1-T')^2}{1+T'}\right)_{00}\right] \tag{20}$$

The quantity $\left(\frac{T'(1-T')}{1+T'}\right)_{00}$ can be evaluated by using the basis of eigenvectors of X' and T', [3, 16, 17]:

$$\left(\frac{T'(1-T')}{1+T'}\right)_{00} = 2\int_0^\infty dk\,(V_0(k))^2\frac{t(k)(1-t(k))}{1+t(k)} + \left(V_0^{(\xi)}V_0^{(\xi)} + V_0^{(\bar{\xi})}V_0^{(\bar{\xi})}\right)\frac{e^{-|\eta|}(1-e^{-|\eta|})}{1+e^{-|\eta|}} \tag{21}$$

The variable k parametrizes the continuous spectrum and $V_0(k)$ is the relevant component of the continuous basis. The modulus 1 numbers ξ and $\bar{\xi}$ parametrize the discrete spectrum and $V_0^{(\xi)}$, $V_0^{(\bar{\xi})}$ are the relevant components of the discrete basis (see [17] for explicit expressions of the eigenvectors and for the connection between ξ, η and b). The discrete spectrum part of the RHS of (21) is just a number. Let us concentrate on the continuous spectrum contribution. We have $t(k) = -\exp(-\frac{\pi|k|}{2})$. Near $k = 0$, $V_0(k) \sim \frac{1}{2}\sqrt{\frac{b}{2\pi}}$ and the integrand $\sim -\frac{b}{2\pi^2}\frac{1}{k}$, therefore the integral diverges logarithmically, a singularity we can regularize with an infrared cutoff ϵ. Taking the signs into account we find that the RHS of (21) goes like $\frac{b}{2\pi^2}\log\epsilon$ as a function of the cutoff. Similarly one can show that $\left(\frac{(1-T')^2}{1+T'}\right)_{00}$ goes like $-\frac{b}{\pi^2}\log\epsilon$. Since for $s \neq s'$, $s^2 + s'^2 > 2ss'$, we can conclude that $\mathcal{C}(s,s') \sim -c\log\epsilon$, where c is a positive number. Therefore, when we remove the cutoff, the factor $e^{-\mathcal{C}(s,s')}$ vanishes, so that (19) becomes a $*$–orthogonality relation. Notice that the above logarithmic singularities in the two pieces in the RHS of (21) neatly cancel each other when $s = s'$ and we get the finite number

$$\mathcal{C}(s,s) = -\frac{s^2}{2b}(1 - S'_{00})$$

In conclusion we can write

$$|\Xi_0'(s)\rangle * |\Xi_0'(s')\rangle = \hat{\delta}(s,s')|\Xi_0'(s)\rangle \tag{22}$$

where $\hat{\delta}$ is the Kronecker (not the Dirac) delta.

Similarly one can prove that

$$\langle\Xi_0'(s')|\Xi_0'(s)\rangle = \frac{\mathcal{N}^2}{\sqrt{\det(1-S'^2)}}\, e^{-\frac{s^2+s'^2}{2b}(1-S'_{00})}\, e^{\frac{1}{2b}\left[(s^2+s'^2)\left(\frac{S'(1-S')}{1+S'}\right)_{00}+2ss'\left(\frac{1-S'}{1+S'}\right)_{00}\right]} \tag{23}$$

We can repeat the same argument as above and conclude that

$$\langle\Xi_0'(s')|\Xi_0'(s)\rangle = \hat{\delta}(s,s')\,\langle\Xi_0'|\Xi_0'\rangle \tag{24}$$

After the above preliminaries, let us consider a sequence $s_1, s_2, \ldots$ of distinct real numbers and the corresponding sequence of displaced D0–branes $|\Xi_0'(s_n)\rangle$. Due to the property (22) also the string state

$$|\Lambda\rangle = \sum_{n=1}^{\infty}|\Xi_0'(s_n)\rangle \tag{25}$$

is a solution to (5): $|\Lambda\rangle * |\Lambda\rangle = |\Lambda\rangle$. To figure out what it represents let us study its space profile. To this end we must sum all the profiles like (18) and then proceed to a numerical evaluation. In order to get a one dimensional object, we render the sequence $s_1, s_2, \ldots$ dense, say, in the positive x–axis so that we can replace the summation with an integral. The relevant integral is

$$\int_0^\infty ds\,\exp\left[-\alpha(x-s)^2 - i\beta(x-s)\right] = \frac{\sqrt{\pi}}{2\sqrt{\alpha}}\left(e^{-\frac{\beta^2}{4\alpha}}\left(1 + \mathrm{Erf}\left(\frac{i\beta}{2\sqrt{\alpha}} + \sqrt{\alpha}x\right)\right)\right) \tag{26}$$

where Erf is the error function and

$$\alpha = \frac{1}{b}\frac{1-S'_{00}}{1+S'_{00}}, \qquad \beta = \frac{2}{\sqrt{b}}\frac{S'_{0m}a^\dagger_m}{1+S'_{00}}$$

Of course (26) is a purely formal expression, but it becomes meaningful in the $\alpha' \to 0$ limit. As usual, [12], we parametrize this limit with a dimensionless parameter ϵ and take $\epsilon \to 0$. Using the results of [3, 12], one can see that $\alpha \sim 1/\epsilon$, $\beta \sim 1/\sqrt{\epsilon}$, so that $\beta/\sqrt{\alpha}$ tends to a finite limit. Therefore, in this limit, we can disregard the first addend in the argument of Erf. Then, up to normalization, the space profile of $|\Lambda\rangle$ is determined by

$$\frac{1}{2}\left(1 + \mathrm{Erf}(\sqrt{\alpha}x)\right) \tag{27}$$

In the limit $\epsilon \to 0$ this factor tends to a step function valued 1 in the positive real x–axis and 0 in the negative one. Of course a similar result can be obtained numerically to any degree of accuracy by using a dense enough discrete $\{s_n\}$ sequence.

Another way of getting the same result is to use the recipe of [12] first on (18). In this way the middle exponential disappears, while the first exponential is regularized by hand (remember that $S'_{00} \to -1$ as $\epsilon \to 0$), so we replace S'_{00} by a parameter $\mathfrak{s}$ and keep it $\neq -1$. Now it is easy to sum over s_n. Again we replace the summation by an integration and see immediately that the space profile becomes the same as (27).

Let us stress that the derivation of the space profile in the low energy regime we have given above is far from rigorous. This is due to the very singular nature of the lump in this limit, first pointed out by [12]. A more satisfactory derivation will be provided in the next section after introducing a background B field.

In summary, the state $|\Lambda\rangle$ is a solution to (5), which represents, in the low energy limit, a one–dimensional object with a constant profile that extends from the origin to infinity in the x–direction. Actually the initial point could be any finite point of the x–axis, and it is not hard to figure out how to construct a configuration that extend from $-\infty$ to $+\infty$. How should we interpret these condensate of D0–branes? In the absence of supersymmetry it is not easy to distinguish between D–strings and F–strings (see, for instance, [18] for a comparison), however in the last section we will provide some evidence that the one–dimensional solutions of the type $|\Lambda\rangle$ can be interpreted as fundamental strings. This kind of objects are very well–known in string theory as classical solutions, [19, 20, 21, 22, 23], see also [24, 25, 26]. For the time being let us notice that, due to (24),

$$\langle\Lambda|\Lambda\rangle = \sum_{n,m=1}^{\infty} \langle \Xi'_0(s_n)|\Xi'_0(s_m)\rangle = \sum_{n=1}^{\infty} \langle \Xi'_0|\Xi'_0\rangle \tag{28}$$

It follows that the energy of the solution is infinite. Such an (unnormalized) infinity is a typical property of fundamental string solutions, see [19].

4 An improved construction

In this section we would like to justify some of the passages utilized in discussing the space profile of the fundamental string solution in section 3. The problems in section 3 are linked to the well–known singularity of the lump space profile, [12], which arises in the low energy limit ($\epsilon \to 0$) and renders some of the manipulations rather slippery. The origin of this singularity is the denominator $1 + S'_{00}$ that appears in many exponentials. Since, when $\epsilon \to 0$, $S'_{00} \to -1$ the exponentials are ill–defined because the series expansions in $1/\epsilon$ are. The best way to regularize them is to introduce a constant background B–field, [30, 31, 32]. The relevant formulas can be found in [13]. For the purpose of this paper we introduce a B field along two space directions, say x and y (our aim is to regularize the solution in the x direction, but, of course, there is no way to avoid involving in the process another space direction).

Let us use the notation x^α with $\alpha = 1, 2$ to denote x, y and let us denote

$$G_{\alpha\beta} = \Delta\delta_{\alpha\beta}, \qquad \Delta = 1 + (2\pi B)^2 \tag{29}$$

the open string metric. As is well–known, as far as lump solutions are concerned, there is an isomorphism of formulas with the ordinary case by which X', S', T' are replaced, respectively, by $\mathcal{X}, \mathcal{S}, \mathcal{T}$, which explicitly depend on B. One should never forget that the latter matrices involve two space directions. We will denote by $|\hat{\Xi}_0\rangle$ the D0–brane solution in the presence of the B field.

Without writing down all the details, let us see the significant changes. Let us replace formula (14) by

$$|\hat{\Xi}_0(\{s^\alpha\})\rangle = e^{-is^\alpha \hat{p}_\alpha}|\hat{\Xi}_0\rangle \tag{30}$$

It satisfies $|\hat{\Xi}_0(s)\rangle * |\hat{\Xi}_0(s)\rangle = |\hat{\Xi}_0(s)\rangle$ and $\langle\hat{\Xi}_0(s)|\hat{\Xi}_0(s)\rangle = \langle\hat{\Xi}_0|\hat{\Xi}_0\rangle$. Instead of (17) we have

$$|\{\hat{x}^\alpha\}\rangle = \left(\frac{2\Delta}{\pi b}\right)^{\frac{1}{2}} \exp\left[\left(-\frac{x^\alpha x^\beta}{b} - i\frac{2}{\sqrt{b}}a_0^{\alpha\dagger}x^\beta + \frac{1}{2}a_0^{\alpha\dagger}a_0^{\beta\dagger}\right)G_{\alpha\beta}\right]|\Omega_b\rangle \tag{31}$$

Next we have

$$\begin{aligned}\langle\{\hat{x}^\alpha\}|\hat{\Xi}_0(s)\rangle \;&=\; \left(\frac{2\Delta}{\pi b}\right)^{\frac{1}{2}} \frac{\hat{\mathcal{N}}}{\sqrt{\det(1+\mathcal{S}_{00})}}\exp\left[-\frac{1}{b}(x^\alpha - s^\alpha)\left(\frac{1-\mathcal{S}_{00}}{1+\mathcal{S}_{00}}\right)_{\alpha\beta}(x^\beta - s^\beta)\right.\\ &\left. - \frac{2i}{\sqrt{b}}(x^\alpha - s^\alpha)(1+\mathcal{S}_{00})_{\alpha\beta}\mathcal{S}_{0m}{}^{\beta}{}_{\gamma}\, a_m^{\gamma\dagger}\right]\exp\left[-\frac{1}{2}a_n^{\alpha\dagger}\mathcal{W}_{nm,\alpha\beta}a_m^{\beta\dagger}\right]|0\rangle\end{aligned} \tag{32}$$

where $\det(1+\mathcal{S}_{00})$ means the determinant of the 2x2 matrix $(1+\mathcal{S}_{00})_{\alpha\beta}$ and

$$\mathcal{W}_{nm,\alpha\beta} = \mathcal{S}_{nm,\alpha\beta} - \mathcal{S}_{n0,\alpha}{}^{\gamma}\left(\frac{1}{1+\mathcal{S}_{00}}\right)_{nm,\gamma\delta}\mathcal{S}_{0m}{}^{\delta}{}_{\beta} \tag{33}$$

The state we start from, i.e. $|\hat{\Xi}_0(s)\rangle$, and the relevant space profile, are obtained by setting $s^1 = s$ and $s^2 = 0$ in the previous formulas.

Next we have an analog of (19) with $\mathcal{C}(s, s')$ replaced by

$$\hat{\mathcal{C}}(s, s') = -\frac{1}{2b}(s^2 + s'^2)\left(\frac{\mathcal{T}(1-\mathcal{T})}{1+\mathcal{T}}\right)_{00,11} - \frac{ss'}{2b}\left(\frac{(1-\mathcal{T})^2}{1+\mathcal{T}}\right)_{00,11} \tag{34}$$

Proceeding in the same way as in section 3 we can prove the analog of eq.(22). By using the spectral representation worked out in [29] one can show that $\hat{\mathcal{C}}$ picks up a logarithmic singularity unless $s = s'$. In a similar way one can prove the analog of (24).

Now let us discuss the properties of

$$|\hat{\Lambda}\rangle = \sum_{n=1}^{\infty} |\hat{\Xi}_0(s_n)\rangle$$

in the low energy limit. We refer to (32) with $s^1 = s$ and $s^2 = 0$. The fundamental difference between this formula and (18) is that in the low energy limit $\mathcal{S}_{00,\alpha\beta}$ becomes diagonal and takes on a value different from -1. More precisely

$$\mathcal{S}_{00,\alpha\beta} \to \frac{2|a|-1}{2|a|+1} G_{\alpha\beta}, \qquad a = -\frac{\pi^2}{V_{00} + \frac{b}{2}} B$$

see [14]. Therefore the $1 + \mathcal{S}_{00}$ denominators in (32) are not dangerous any more. Similarly one can prove that in the same limit $\mathcal{S}_{0n} \to 0$. Moreover the ϵ–expansions about these values are well–defined. Therefore the space profile we are interested in is

$$\sim \exp[-\frac{\mu}{b}(x+s)^2 - \frac{\mu}{b}(y)^2] \exp[-\frac{1}{2} a_n^{\alpha\dagger} \mathcal{S}_{nm,\alpha\beta} a_m^{\beta\dagger}]|0\rangle \tag{35}$$

with a finite normalization factor and $\mu = \frac{2|a|-1}{2|a|+1}\Delta$. Now one can safely integrate s and obtain the result illustrated in section 3. This also sheds light on how the resulting state couples to the $B_{\mu\nu}$ field. Indeed the length of this one dimensional objects is measured with the open string metric (29), in other words the B–field couples to the string by "stretching" it.

5 Fundamental strings

In this section we would like to discuss the properties of the Λ solutions we found in the previous sections. In order to justify the claim we made that they represent fundamental strings, in the sequel we show that they are still solutions if we attach them to a D–brane. To this end let us pick $|\Lambda\rangle$ as given by (25) with $s_n > 0$ for all n's. Now let us consider a D24–brane with the only transverse direction coinciding with the x–axis and centered at $x = 0$. The corresponding lump solution has been introduced at the end of section 2 (case $k = 24$). Let us call it $|\Xi'_{24}\rangle$. Due to the particular configuration chosen, it is easy to prove that $|\Xi'_{24}\rangle + |\Lambda\rangle$ is still a solution to (5). This is due to the fact that $|\Xi'_{24}\rangle$ is $*$-orthogonal to the states $|\Xi'_0(s_n)\rangle$ for all n's. To be even more explicit we can study the space profile of $|\Xi'_{24}\rangle + |\Lambda\rangle$, assuming the sequence s_n to become dense in the positive x–axis. Using the previous results it is not hard to see that the overall configuration is a Gaussian centered at $x = 0$ in the x direction (the D24-brane) with an infinite prong attached to it and extending along the positive x–axis. The latter has a Gaussian profile in all space directions except x.

We remark that the condition $s_n > 0$ for all n's is important because $|\Xi'_{24}\rangle + |\Lambda\rangle$ is not anymore a projector if the $\{s_n\}$ sequence contains 0, since $|\Xi'_0(0)\rangle$ is not $*$–orthogonal to $|\Xi'_{24}\rangle$. This remark tells us that it not possible to have solutions representing configurations in which the string crosses the brane by a finite amount: the string has to stop at the brane.

This is to be contrasted with the configuration obtained by replacing $|\Lambda\rangle$ in $|\Xi'_{24}\rangle + |\Lambda\rangle$ with a D1–brane along the x axis, that is with $|\Xi'_1\rangle$. The state we get is definitely not a solution to the (5). This of course reinforces the interpretation of the $|\Lambda\rangle$ solution as a fundamental string.

Needless to say it is trivial to generalize the solution of the type $|\Xi'_{24}\rangle + |\Lambda\rangle$ to lower dimensional branes.

It is worth pointing out that it is also possible to construct string solutions of finite length. It is enough to choose the sequence $\{s_n\}$ to lie between two fixed values, say a and b in the x–axis, and then 'condense' the sequence between these two points. In the low energy limit the resulting solution shows precisely a flat profile for $a < x < b$ and a vanishing profile outside this interval (and of course a Gaussian profile along the other space direction). This solution is fit to represent a string stretched between two D–branes located at $x = a$ and $x = b$.

An important property for fundamental strings is the exchange property. Let us see if it holds for our solutions in a simple example. We consider first an extension of the solution (25) made of two pieces at right angles. Let us pick two space directions, x and y. We will denote by $\{s^x_n\}$ and $\{s^y_n\}$ a sequence of points along the positive x and y–axis. The string state

$$|\Lambda^{\pm\pm}\rangle = |\Xi'_0\rangle + \sum_{n=1}^{\infty} |\Xi'_0(\pm s^x_n)\rangle + \sum_{n=1}^{\infty} |\Xi'_0(\pm s^y_n)\rangle \tag{36}$$

is a solution to (5). The $\pm\pm$ label refers to the positive (negative) x and y–axis. This state represents an infinite string stretched along the positive (negative) x and y–axis including the origin. Now let us construct the string state

$$|\Xi'_0\rangle + \sum_{n=1}^{\infty} |\Xi'_0(s^x_n)\rangle + \sum_{n=1}^{\infty} |\Xi'_0(-s^x_n)\rangle + \sum_{n=1}^{\infty} |\Xi'_0(s^y_n)\rangle + \sum_{n=1}^{\infty} |\Xi'_0(-s^y_n)\rangle \tag{37}$$

This is still a solution to (5) and can be interpreted in two ways: either as $|\Lambda^{++}\rangle + |\Lambda^{--}\rangle$ or as $|\Lambda^{+-}\rangle + |\Lambda^{-+}\rangle$, up to addition to both of $|\Xi'_0\rangle$ (a bit removed from the origin). This addition costs the same amount of energy in the two cases, an amount that vanishes in the continuous limit. Therefore the solution (37) represents precisely the exchange property of fundamental strings.

So far we have considered only straight one–dimensional solutions (in terms of space profiles), or at most solutions represented by straight lines at right angles. However this is an unnecessary limitation. It is easy to generalize our construction to any curve in space. For instance, let us consider two directions in space and let us denote them again x and y ($\hat{p}^x$ and $\hat{p}^y$ being the relevant momentum operators). Let us construct the state

$$|\Xi'_0(s^x, s^y)\rangle = e^{-is^x\hat{p}^x} e^{-is^y\hat{p}^y} |\Xi'_0\rangle \tag{38}$$

It is evident that this represents a space–localized solution displaced from the origin by s^x in the positive x direction and s^y in the positive y direction. Using a suitable sequence $\{s^x_n\}$ and $\{s^y_n\}$, and rendering it dense, we can construct any curve in the $x - y$ plane, and, as a consequence, write down a solution to the equation of motion corresponding to this curve. The generalization to other space dimensions is straightforward.

We would like to remark that, by generalizing the above construction, one can also construct higher dimensional objects. For instance one could repeat the accretion construction by adding parallel D1–branes (that extend, say, in the y direction) along the x–axis. In this way we end up with a membrane-like configuration (with a flat profile in the x, y–plane), and continue in the same tune with higher dimensional configurations.

All the solutions we have considered so far are unstable. However the fundamental string solutions are endowed with a particular property. Since they end on a D–brane, their endpoints couple to the electromagnetic field on the brane, [19, 27, 28], and carry the corresponding charge. When the D–brane decays there is nothing that prevents the (fundamental) strings attached to it from decaying themselves. However in the presence of a background E–field, the latter are excited by the coupling with the E–field and persist (or, at least, persist longer than the other unstable objects). This phenomenon is described in [23] in effective field theory and BCFT language (see also [29]).

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