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The world-sheet corrections to dyons in the Heterotic theory

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Abstract: All the linear α' corrections in the compactification of the critical Heterotic string theory on T^6 are computed for a BPS static spherical four dimensional dyonic black hole representing a wrapped fundamental string carrying arbitrary winding and momentum charges along one cycle in the presence of KK-monopole and H-monopole charges associated to another cycle. It is showed that the corrections to the modified Hawking-Bekenstein entropy can not be reproduced by the inclusion of only the Gauss-Bonnet Lagrangian to the supergravity approximation of the Lagrangian for dyons.

Keywords: Black holes in string theory, Heterotic strings, Sigma model, Horizon, Black holes.

Contents

1. Introduction

Dyonic black holes are black holes which carry electric charges and magnetic charges of some gauge fields. Some of the dyonic black holes can be realised as the solutions of the supergravity approximation to the critical Heterotic string theory compactified on T^6. There exists a proposal for the exact degeneracy of microsates of dyons in torodially compactified critical Heterotic string theory [1, 2, 3, 4, 5, 6]. The logarithm of the degeneracy of dyons defines the statistical entropy.

At the supergravity approximation the Hawking-Bekenstein entropy is in agreement with the large-charge-limit of the statistical entropy. The dominant string corrections to the dyons are the α' corrections. Thus the α' corrections to the thermodynamical entropy for the dyonic black hole should be in agreement with the large charge expansion series of the statistical entropy. Ref [7, 8, 9, 10, 11, 12, 13, 14] observed that upon the inclusion of the square of the Riemann tensor and a supersymmetric completion of that, the modified Hawking-Bekenstein entropy [15, 16] is in agreement with the statistical entropy. Ref [17] showed that the inclusion of the Gauss-Bonnet action gives the same corrections in modified Hawking-Bekenstein entropy as those given by the inclusion of supersymmetric version of the square of the Riemann tensor.

The Gauss-Bonnet Lagrangian or the supersymmetric version of square of the Riemann tensor are not all the linear α' corrections to the dyonic black holes. It is remained unanswered why other linear α' terms should not contribute to the modified Hawking-Bekenstein entropy. In this note we consider a BPS static spherical four dimensional dyonic black hole representing a wrapped fundamental string carrying arbitrary winding and momentum charges along one cycle in the presence of KK-monopole and H-monopole charges associated to another cycle. Then we compute all the linear α' corrections in the modified Hawking-Bekenstein entropy [15, 16] for this dyon. The note is organised in the following way:

In the second section we consider the Low Energy Effective Action of the Heterotic string theory. We study a KK-compactification of the Heterotic string theory on T^6 relevent for a BPS static spherical four dimensional dyonic black hole representing a wrapped fundamental string carrying arbitrary winding and momentum charges along one cycle in the presence of KK-monopole and H-monopole charges of another cycle [18]. We show that the pull back of the ten dimensional perturbative LEEA action to the four dimensions, the induced action, is expressible in a covariant form in terms of the Riemann tensor constructed from the four dimensional metric, the exterior derivatives of the form fields, the scalars and their covariant derivatives respect to the four dimensional metric.

In the third section we present all the linear α' corrections in the heterotic string to ten dimensional backgrounds composed of the metric, the NS two-form and the dilaton derived from string amplitude considerations on sphere [19, 20]. We apply the compactification process of the second section to account for all the linear α' corrections in the induced action. We study the α' corrections as perturbations outside the horizon. We notice that for a general black hole requiring a smooth α' perturbation on the horizon may alter the charges of the black hole. The attractor equations and the entropy formalism do not answer if (and how much) the charges are corrected. Therefore generically the attractor equations [21, 22] and the entropy formalism [23, 24] do not suffice to express the parameters of the horizon configuration in terms of the values of the charges at the supergravity approximation. We show that the charges of a dyonic black hole retain their values at the supergravity approximation because there exists a scheme in which the fields at the supergravity approximation do not receive any α' corrections [25, 26]. Therefore one may employ the entropy formalism to compute the linear α' corrections to the entropy of a dyonic black hole.

In the fourth section we evaluate the induced action near the horizon configuration. We employ the attractor mechanism [21, 22] and the entropy formalism [23, 24] to calculate the modified Hawking-Bekenstein. We will see that all the linear α' corrections to the entropy of a regular dyon with a large horizon can not be reproduced by inclusion of only the Gauss-Bonnet Lagrangian to the Lagrangian density at the supergravity approximation. For the case of regular dyons with a large event horizon this suggests that the map from the index defined in [1]-[6] to the thermodynamical entropy should be modified at the linear order in α'.

In the fourth section we summarise and discuss the results.

2. The compactification of the LEEA

We consider a ten-dimensional Riemannian manifold $\boldsymbol{M}_{10}$ homeomorphic to $M_4 \times T^6$ whose metric has six killing vectors in T^6 and admits an asymptotically flat region. We represent the coordinate patch that covers the asymptotic flat region of $\boldsymbol{M}_{10}$ by $\boldsymbol{x}^i$ =$(t,\ x^1,\ x^2,\ x^3,\ y_1,\ y_2,\ z_1,\ \cdots,\ z_4)$ where $x^\mu = (t, \cdots, x^3)$ and $(y^m, z^m) = (y_1, y_2, z_1, \cdots, z_4)$ are coordinates respectively on M_4 and T^6 and $(dy_1, \cdots, dz_4)$ are the killing vectors. We refer to x_μ and $(y^m$,$z^m)$ respectively as the four dimensions and the compactified space. The string perturbations can be studied at the vicinity of the asymptotic region of $\boldsymbol{M}_{10}$ which is covered by $\boldsymbol{x}^i$. We realise this neighbourhood as a background of the Heterotic string theory composed of the metric, the NS two-form and the dilaton whose field configuration follows

$$\begin{aligned}
\boldsymbol{ds^2} &= \sum_{\mu,\nu=1}^{4} \boldsymbol{g}_{\mu\nu}(x)dx^\mu dx^\nu + \sum_{m=1}^{2}\{2\boldsymbol{g}_{y_m\mu}(x)\, dy_m\, dx^\mu + \boldsymbol{g}_{y_m y_m}(x)dy_m^2\} + \sum_{m=1}^{4} dz_m^2\,, \\
\boldsymbol{B} &= \boldsymbol{B}_{\mu\nu}(x)dx^\mu \wedge dx^\nu + \boldsymbol{B}_{y_1\mu}(x)dx^\mu \wedge dy_1 + \boldsymbol{B}_{y_2\mu}(x)dx^\mu \wedge dy_2\,, \qquad (2.1) \\
\boldsymbol{\phi} &= \boldsymbol{\phi}(x)\,, \qquad (2.2)
\end{aligned}$$

We use the bold symbols to represent the fields in ten dimensions. We rewrite the metric in the following form

$$\boldsymbol{ds}^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu + T_1(x)^2(dy_1 + 2A^1_\mu(x)dx^\mu)^2 + T_2(x)^2(dy_2 + 2A^2_\mu(x)dx^\mu)^2 + dz_m^2\,, \qquad (2.3)$$

where $g_{\mu\nu}$, T_1, T_2, A^1_μ and A^2_μ are used to re-express the components of the ten dimensional metric in a way that we shall see is more convenient. The metric (2.3) is invariant under the following transformations

$$\begin{cases} y_1 \to y_1 - 2\Lambda^1(x) \\ A^1_\mu(x) \to A^1_\mu(x) + \partial_\mu \Lambda^1(x) \end{cases} , \begin{cases} y_2 \to y_2 - 2\Lambda^2(x) \\ A^2_\mu(x) \to A^2_\mu(x) + \partial_\mu \Lambda^2(x) \end{cases} \qquad (2.4)$$

where $\Lambda^1(x)$ and $\Lambda^2(x)$ are arbitrary scalars. These symmetries are remnants of the ten-dimensional diffeomorphism. We interpret $A^1_\mu(x)$ and $A^2_\mu(x)$ as two distinct $U(1)$ gauge connections in the four dimensions because they are vectors and they have $U(1)$ symmetries associated to them.

We rewrite the NS two-form in the following way

$$\boldsymbol{B} = B_{\mu\nu}dx^\mu \wedge dx^\nu + 2A^3_\mu dx^\mu \wedge (dy_1 + 2A^1_\nu dx^\nu) + 2A^4_\mu dx^\mu \wedge (dy_2 + 2A^2_\nu dx^\nu)\,, \quad (2.5)$$

where $B_{\mu\nu}$, A^3_μ and A^4_μ are used to re-express the components of the ten dimensional NS two-form in a way that we shall see is more convenient. Note that the $U(1)$ transformations associated to $A^1_\mu(x)$ and $A^2_\mu(x)$ leave intact $B_{\mu\nu}(x)$, $A^3(x)$ and $A^4(x)$ since $dy_i + 2A^i_\nu dx^\nu$ is gauge invariant. The sigma model for the background we are considering is invariant under altering $\boldsymbol{B}$ by any exact two-form, i.e.

$$\boldsymbol{B} \to \boldsymbol{B} + \boldsymbol{d\Lambda}\,, \qquad (2.6)$$

and thus the low energy action is invariant under $\boldsymbol{B} \rightarrow \boldsymbol{B} + \boldsymbol{d\Lambda}$. Amongst these Λ's we consider the ones given by

$$\boldsymbol{\Lambda} = \Lambda_\mu(x)dx^\mu - \Lambda^3(x)dy_1 - \Lambda^4(x)dy_2\,,$$

which imply that the LEEA is invariant under

$$B \rightarrow B + d\Lambda\,, \tag{2.7}$$
$$A^3_\mu(x) \rightarrow A^3_\mu(x) + \partial_\mu\Lambda^3(x)\,, \tag{2.8}$$
$$A^4_\mu(x) \rightarrow A^4_\mu(x) + \partial_\mu\Lambda^4(x)\,. \tag{2.9}$$

We see that independent $U(1)$ symmetries are associated to $A^3_\mu(x)$ and $A^4_\mu(x)$. These $U(1)$ symmetries are remnants of the gauge symmetries in ten-dimensions. We interpret $A^3_\mu(x)$ and $A^4_\mu(x)$ as two distinct gauge connections in the four dimensions.

Due to the symmetries of the metric we can choose a sufficiently large volume for any non-trivial cycle[1] in the compactified space in the patch of $\boldsymbol{x}^i$. Thus we ignore the world-sheet or target space instantons corrections to the LEEA. The rest of the string corrections respect the ten dimensional diffeomorphism symmetry group. Therefore the ten-dimensional low energy effective action reads

$$\boldsymbol{S} \;=\; \frac{1}{32\,\pi}\int d^{10}\boldsymbol{x}\sqrt{-\det\boldsymbol{g}}\,e^{-2\boldsymbol{\phi}}\,\boldsymbol{L}(\boldsymbol{B},\boldsymbol{g},\boldsymbol{\phi})\,, \tag{2.10}$$

where $\boldsymbol{L}(\boldsymbol{B},\boldsymbol{g},\boldsymbol{\phi})$ includes all the perturbative string corrections and it is invariant under ten-dimensional diffeomorphism and $\boldsymbol{B} \rightarrow \boldsymbol{B} + \boldsymbol{d\Lambda}$. Inserting (2.3) and (2.5) into the action (2.10) we obtain

$$S \;=\; \frac{1}{32\,\pi}\int d^4x\,\sqrt{-\det g}\,e^{-2\phi}\,L(g_{\mu\nu}, B_{\mu\nu}, A^1_\mu, \cdots, A^4_\mu, T_1, T_2, \phi)\,, \tag{2.11}$$

where the integration on the compactified space is performed and we have defined

$$2\phi = 2\boldsymbol{\phi} - \ln T_1 - \ln T_2 - \ln V\,, \tag{2.12}$$

where V is the volume of the compactified space. (2.11) is the pullback of the action into the four dimensions and we refer to it as the induced action. The induced action inherits the remnants symmetries of the ten dimensional action. Thus it is invariant under four dimensional diffeomorphism group, four $U(1)$ symmetries associated to $A^1_\mu, \cdots, A^4_\mu$ and $B \rightarrow B + d\Lambda$. This means that the induced Lagrangian is expressible in a covariant form in terms of the Riemann tensor constructed from $g_{\mu\nu}$, the exterior derivatives of the form fields, T_1, T_2, ϕ and their covariant derivatives respect to $g_{\mu\nu}$,

$$L = L(R_{\mu\nu\lambda\eta}, dB, dA^1, \cdots, dA^4, T_1, T_2, \phi, g_{\mu\nu}, \nabla_\mu)\,\blacksquare \tag{2.13}$$

[1]A cycle which does not shrink to a point under any given homeomorphism.

3. Dyonic black holes and the α' corrections

In the supergravity approximation to the critical heterotic string theory the action for backgrounds of the metric ($\boldsymbol{g}$), the NS two-form ($\boldsymbol{B}$) and the dilaton ($\boldsymbol{\phi}$) reads

$$\boldsymbol{S}^{(0)} = \frac{1}{32\pi}\int d^{10}\boldsymbol{x}\,\sqrt{-\det\boldsymbol{g}}\,e^{-2\boldsymbol{\phi}}\,(\boldsymbol{R}_{\rm Ricci} + 4|\boldsymbol{\nabla\phi}|^2 - \frac{1}{12}\boldsymbol{H}_{ijk}\boldsymbol{H}^{ijk})\,, \qquad (3.1)$$

where

$$\boldsymbol{H}_{ijk} = 3\,\boldsymbol{B}_{[ij,k]}\,. \qquad (3.2)$$

The linear α' corrections to this action derived from string amplitude considerations on sphere [20] read

$$\boldsymbol{S}^{(1)} = \frac{1}{32\pi}\int d^{10}\boldsymbol{x}\sqrt{-\det\boldsymbol{g}}\;e^{-2\boldsymbol{\phi}}\,\frac{\alpha'}{8}\,\Big(\boldsymbol{R}_{klmn}\boldsymbol{R}^{klmn} - \frac{1}{2}\boldsymbol{R}_{klmn}\boldsymbol{H}_p{}^{kl}\boldsymbol{H}^{pmn}$$
$$-\frac{1}{8}\boldsymbol{H}_k{}^{mn}\boldsymbol{H}_{lmn}\boldsymbol{H}^{kpq}\boldsymbol{H}^l{}_{pq} + \frac{1}{24}\boldsymbol{H}_{klm}\boldsymbol{H}^k{}_{pq}\boldsymbol{H}_r{}^{lp}\boldsymbol{H}^{rmq}\Big)\,. \qquad (3.3)$$

Because of the consistency of the Heterotic string around flat space-time (3.3) should be in agreement with worldsheet-loop calculations of the sigma model. Note that ref. [20] computed the linear α' corrections for the Heterotic and the Bosonic string theory. Eq. (3.3) is eq. (3.24) of ref. [20] for $\lambda_0 = \frac{1}{8}$. The corrections given by (3.3) are all four derivative corrections to the action required by string amplitude considerations on sphere. Thus (3.3) is compatible with anomaly cancellation [27] for backgrounds of vanishing $\mathrm{Spin}(32)/Z_2$ or $E8\times E8$ connection up to the field redefinition ambiguities.

We compactify the ten dimensional background to the four dimensions in the way that we presented in the previous section. This means that we rewrite the ten dimensional background by (2.3), (2.5) and (2.12) in terms of the four dimensional background fields; the metric $g_{\mu\nu}$, the two-form $B_{\mu\nu}$, the four $U(1)$ gauge connections $A^1\cdots A^4$ and the three scalars T_1, T_2 and ϕ. It is not a hard task to obtain the explicit form of the induced action at the level of supergravity approximation,

$$S^{(0)} = \frac{1}{32\pi}\int d^4x\,\sqrt{-\det g}\,e^{-2\phi}\left(R - \frac{|dB|^2}{12} + 4|\nabla\phi|^2 - |\nabla\ln T_1|^2 - |\nabla\ln T_2|^2\right.$$
$$\left.-|T_1\,dA^1|^2 - |T_2\,dA^2|^2 - |\frac{dA^3}{T_1}|^2 - |\frac{dA^4}{T_2}|^2\right)\,, \qquad (3.4)$$

where R denotes the Ricci scalar of $g_{\mu\nu}$ and integrations by parts are understood. We do not obtain the explicit form of the linear α' corrections to the induced action. We suffice to present the linear α' corrections to the induced action by

$$S^{(1)} = \frac{1}{32\,\pi}\int d^4x\,\sqrt{-\det g}\,e^{-2\phi}\,L^{(1)}\,, \qquad (3.5)$$

and we know that $L^{(1)}$ is a functional of the four dimensional Riemann tensor, the gauge field strengths and their covariant derivatives,

$$L^{(1)} = L^{(1)}(R_{\mu\nu\lambda\eta}, dB, dA^1, \cdots, dA^4, T_1, T_2, \phi, g_{\mu\nu}, \nabla_\mu)\,. \tag{3.6}$$

At the level of the four dimensional supergravity approximation we consider a static spherical dyonic black hole which carries electric charges of A^1 and A^3 and magnetic charges of A^2 and A^4 in an asymptotically flat space-time. The electric charges of A^1 and A^3 represent respectively the KK-momentum and winding numbers of a fundamental string wrapped around the cycle of y_1. The magnetic charges of A^2 and A^4 represent respectively the KK-monopole and the H-monopole charges associated to the cycle of y_2. The explicit forms of the fields for this dyonic black hole are presented in [18]. When none of the charges is zero then the dyonic black hole has a regular horizon with geometry of $AdS_2 \times S^2$ outside which the string loop corrections can be ignored. At the supergravity approximation the $SO(6,22) \times SL(2,Z)$ duality transformations can be applied on the dyonic black hole to obtain a general dyonic black hole [28]. Reminiscent to T-duality and α' corrections [29, 30, 31], we expect that the duality transformations themselves get modified by the α' corrections. In this note we do not use the duality transformations. We consider a dyonic black hole with large momentum, winding, KK-monopole and H-monopole charges and we study the α' corrections as perturbations on and outside its horizon.

We use Ψ_i to represent all the fields of the dyonic black hole in a collective fashion[2],

$$\Psi_i \in \left\{g_{\mu\nu}(x), T_1(x), T_2(x), A^1_\mu(x), \cdots, A^4_\mu(x), \phi(x)\right\}\,. \tag{3.7}$$

The action for this collective notation follows

$$S[\Psi] = S^0[\Psi] + \alpha' S^1[\Psi] + O(\alpha'^2)\,, \tag{3.8}$$

the equations of motion of which read

$$0 \;=\; \frac{\delta S[\Psi]}{\delta \Psi_i} = \frac{\delta S^0[\Psi]}{\delta \Psi_i} + \alpha' \frac{\delta S^1[\Psi]}{\delta \Psi_i} + O(\alpha'^2)\,, \tag{3.9}$$

where $\frac{\delta}{\delta\Psi_i}$ stands for the functional derivative respect to Ψ_i. We write an α' expansion series for Ψ_i and we solve (3.9) perturbatively,

$$\Psi_i = \Psi^0_i + \alpha' \Psi^1_i + O(\alpha'^2)\,. \tag{3.10}$$

Inserting this perturbative expansion in (3.9) gives

$$\frac{\delta S^0[\Psi]}{\delta \Psi_i}\Big|_{\alpha'=0} + \alpha'\Big(\frac{\delta^2 S^0[\Psi]}{\delta \Psi_i \delta \Psi_j}\Big|_{\alpha'=0} \Psi^1_j + \frac{\delta S^1[\Psi]}{\delta \Psi_i}\Big|_{\alpha'=0}\Big) + O(\alpha'^2) \;=\; 0\,, \tag{3.11}$$

[2]This collective notation is constructed in analogy with the compact notation used in [32].

which implies

$$\frac{\delta S^0[\Psi]}{\delta \Psi_i}\Big|_{\alpha'=0} = 0\,, \tag{3.12}$$

$$\frac{\delta^2 S^0[\Psi]}{\delta \Psi_i \delta \Psi_j}\Big|_{\alpha'=0} \Psi_j^1 = -\frac{\delta S^1[\Psi]}{\delta \Psi_i}\Big|_{\alpha'=0}\,. \tag{3.13}$$

Note that (3.12) stands for the equations of motion at the supergravity approximation and (3.13) gives a set of non-homogeneous linear second order differential equations for $\{\Psi_i^1\}$ for any given solution at the supergravity approximation $\{\Psi_i^0\}$.

Let us first study the solutions to the homogeneous equations which correspond to (3.13),

$$\frac{\delta^2 S^0[\Psi]}{\delta \Psi_i \delta \Psi_j}\Big|_{\alpha'=0} \Psi_{j,B}^1 = 0\,. \tag{3.14}$$

These are the equations describing the fluctuations around $\Psi = \Psi_0$ at the supergravity approximation. For the dyonic black hole, the equation for a static spherical fluctuation of the dilaton in the canonical frame is simplified to

$$\partial_r((r - r_H)^2 \partial \phi_B^1) = 0\,, \tag{3.15}$$

where r_H is the radius of the horizon and ϕ_B^1 is the fluctuation of the dilaton and we have used the explicit form of the background fields presented in [18]. The general solution of (3.15) is diverging on the horizon,

$$\phi_B^1(r) = \frac{c_1}{r - r_H} + c_2\,. \tag{3.16}$$

The diverging mode of the dilaton fluctuations plays the role of the diverging source for the fluctuations of all other fields through their couplings to the dilaton. Thus the fluctuations of all other fields admit modes which diverge on the horizon. Therefore we conclude that

> **Lemma 1:** The general solutions to the homogeneous equations (3.14) diverge on the horizon.

The diverging solutions on the horizon should be excluded by the boundary conditions. We impose the following boundary conditions on the solutions of (3.13)

$$\begin{cases} \Psi_i^1(x)\big|_{x=\infty} = 0\,, \\ \Psi_i^1(x)\big|_{x \text{ on the Horizon}} < \infty\,, \end{cases} \tag{3.17}$$

we refer to which as the H-boundary conditions. The first condition of the H-boundary conditions set the α' corrections to zero at infinity and its second condition excludes the diverging modes on the horizon. Depends on how we decide to represent the metric, some of the components of the metric may diverge on the horizon at the supergravity approximation. For these components of the metric we substitute the second condition of the H-boundary conditions by

$$\lim_{x\to\Sigma_h} \frac{\Psi_i^1(x)}{\Psi_i^0(x)} < \infty\,, \tag{3.18}$$

where Σ_h represents any point on the horizon. Because the α' corrections reaches their largest values on the horizon then having fixed the symmetries the H-boundary conditions guaranty that $\Psi_i^1(x)$ is bounded outside the horizon,

$$\Psi_i^1(x) < \infty\ ,\ \forall |x| \in [r_H, \infty)\,. \tag{3.19}$$

Second order linear differential equations have two solutions. In general the H-boundary conditions exclude one of the solutions and identify the other one. There exits no further freedom to impose more constraints on the solutions. Thus we conclude that:

> **Lemma 2:** The H-boundary conditions do not necessarily retain the fall off of the fields at asymptotic infinity.

Note that these lemmas are not in contradiction with supersymmetry. If we knew the α' corrections to the supersymmetric constraints then we could have used the supersymmetric constraints rather the equations of motions to obtain a set of non-homogeneous first order linear differential equations for the α' corrections to the background fields. Requiring the α' corrections to vanish at infinity fixes all the boundary conditions for these first order equations. Thus again we conclude that the fall off of the fields at asymptotic infinity might receive α' corrections. In addition we learn that the diverging modes on the horizon are non-supersymmetric fluctuations on the supersymmetric background.

For the Schwarzschild black hole, it is showed [29, 33] that imposing the H-boundary conditions produces corrections to the Newtonian mass of the black hole which is given by the fall off of the time-time component of the canonical metric at asymptotic infinity. The fall off of the fields identifies the charges of the dyonic black hole. Thus the second lemma implies that the charges of the dyonic black hole might get modified by the α' corrections.

In the perturbative study of the string scattering amplitudes one is allowed to redefine the fields,

$$\tilde{\Psi}_i = \Psi_i + \alpha' R_i + O(\alpha'^2) \tag{3.20}$$

where R_i are tensors of appropriate degree and dimension constructed from polynomials of Ψ_i^0 and their derivatives. The field redefinition alters the induced action and subsequently the equations of motion derived from the action. For example a general field redefinition given by (3.20) changes the equations for the linear α' corrections (3.13) to,

$$\frac{\delta^2 S^0[\Psi]}{\delta\Psi_i\delta\Psi_j}\Big|_{\alpha'=0}\tilde{\Psi}^1_j = -\frac{\delta S^1[\Psi]}{\delta\Psi_i}\Big|_{\alpha'=0} - \frac{\delta^2 S^0[\Psi]}{\delta\Psi_i\delta\Psi_j}\Big|_{\alpha'=0} R_i\,. \tag{3.21}$$

The field redefinition ambiguity is related to the freedom in choosing different renormalisation and regularisation schemes in the sigma model. Ref. [25] has considered the dyonic black as a generalisation of the null chiral sigma models [26] and has proved that there exists a scheme in which Ψ^0_i does not receive any α' corrections. This means that there exists $R_i = R_i{}^\star$ for which the right hand side of (3.21) vanishes and $\tilde{\Psi}^1_j = 0$ is the solution to (3.21). Thus the solution in the scheme where the α' corrections are given by (3.13) reads

$$\Psi_i = \Psi^0_i \,-\, \alpha'\, R_i{}^\star \,+\, O(\alpha'^2)\,, \tag{3.22}$$

Since any field redefinition should contain two derivatives then the fall off of Ψ_i at infinity relevant for the charge identification is the same as the one of Ψ^0_i. We conclude that

> **Lemma 3:** There exists no α' correction to the charges of the dyonic black hole.

4. The α' corrections to the entropy of dyons

The near horizon configuration of the dyonic black hole at the supergravity approximation is $AdS_2 \times S^2$. When the horizon is large the α' corrections do not change the geometry of the horizon. Therefore the near horizon configuration of the α' corrected dyonic black hole can be written in the following way

$$ds^2 = v_1(-r^2 d\tau^2 + \frac{dr^2}{r^2}) + v_2(d\theta^2 + \sin^2\theta d\phi^2)\,, \tag{4.1}$$

$$T_1 = T_1 \;,\; T_2 = T_2 \;,\; e^{-2\Phi} = s\,, \tag{4.2}$$

$$F^1_{r\tau} = e_1 \;,\; F^3_{r\tau} = e_3 \;,\; F^2_{\theta\phi} = \frac{p_2}{4\pi}\sin\theta \;,\; F^4_{\theta\phi} = \frac{p_4}{4\pi}\sin\theta\,, \tag{4.3}$$

where the horizon in located at $r = 0$ and $v_1, \cdots, p_4$ are constant parameters labelling the horizon. Note that v_1 and v_2 are constant due to the geometry of the horizon and T_1, T_2, s are constant since they represent the limit of $r \to 0$ of the scalars. e_1, e_3, p_2 and p_4 are constant due to the coordinates chosen to represent the background and in accordance with the supergravity approximation.

In the second section it was proved that the induced action is a functional of the gauge field strengths but not of the gauge fields themselves. Thus the entropy formalism techniques [23, 24] can be employed to express the parameters of the near horizon configuration in terms of the charges of the dyonic black hole. The entropy formalism uses the entropy function defined by

$$f(\vec{v}, \vec{T}, \vec{e}, \vec{p}, s) = \frac{1}{32\,\pi} \int d\theta d\phi \sqrt{-\det g}\, s\, L(\vec{v}, \vec{T}, \vec{e}, \vec{p})\,, \tag{4.4}$$

where $L(\vec{v}, \vec{T}, \vec{e}, \vec{p})$ is the induced Lagrangian (2.13) evaluated on the horizon configuration. The equations of motions are equivalent to

$$\begin{aligned}
\frac{\partial f(\vec{v}, \vec{T}, \vec{e}, \vec{p}, s)}{\partial v_i} &= 0\,, \\
\frac{\partial f(\vec{v}, \vec{T}, \vec{e}, \vec{p}, s)}{\partial T_i} &= 0\,, \\
\frac{\partial f(\vec{v}, \vec{T}, \vec{e}, \vec{p}, s)}{\partial s} &= 0\,, \\
\frac{\partial f(\vec{v}, \vec{T}, \vec{e}, \vec{p}, s)}{\partial e_i} &= q_i\ ,\ \ i \in \{1, 3\}
\end{aligned} \tag{4.5}$$

where q_1 and q_3 are the electric charges of the dyonic black hole and p_2 and p_4 are identified as the magnetic charges. In the following we first obtain these equations and next we find their α' perturbative solutions.

To evaluate the induced action on the horizon configuration we use (2.3), (2.5) and (2.12) to write the ten dimensional fields for the near horizon configuration,

$$d\boldsymbol{s}^2 = ds^2 + T_1(dy_1 + 2\, e_1\, r\, dt)^2 + T_2(dy_2 - \frac{p_2}{2\pi}\cos\theta\, d\phi)^2 + dz_m^2\,, \tag{4.6}$$

$$\boldsymbol{B} = 2\, e_3\, r\, dt \wedge dy_1 - \frac{p_4}{2\pi}\cos\theta\, d\phi \wedge dy_2\,, \tag{4.7}$$

$$e^{-2\boldsymbol{\phi}} = \frac{s}{V\ T_1\ T_2}\,, \tag{4.8}$$

where V is the volume of the compactified space and the gauges (4.3) are chosen by

$$A^1_\mu = [e_1\, r, 0, 0, 0]\ ,\ A^3_\mu = [e_3\, r, 0, 0, 0]\,, \tag{4.9}$$

$$A^2_\mu = [0, 0, 0, -\frac{p_2}{4\pi}\cos\theta]\ ,\ A^4_\mu = [0, 0, 0, -\frac{p_4}{4\pi}\cos\theta]\,. \tag{4.10}$$

In ten dimensions using the ten dimensional fields (4.6) and (4.7) one finds that

$$\begin{aligned}
L_0 &= \boldsymbol{R}_{\text{Ricci}} - \frac{1}{12}\boldsymbol{H}_{ijk}\boldsymbol{H}^{ijk} = \\
&= -\frac{2}{v_1} + \frac{2}{v_2} + \frac{2{e_1}^2{T_1}^2}{{v_1}^2} + \frac{2{e_3}^2}{{v_1}^2{T_1}^2} - \frac{{p_2}^2{T_2}^2}{8{v_2}^2\pi^2} - \frac{{p_4}^2}{8{v_2}^2\pi^2{T_2}^2},
\end{aligned} \tag{4.11}$$

$$L_1 = \frac{1}{8}\boldsymbol{R}_{klmn}\boldsymbol{R}^{klmn} = \tag{4.12}$$

$$= +\frac{1}{2\,{v_1}^2} + \frac{1}{2\,{v_2}^2} - \frac{3{e_1}^2\,{T_1}^2}{{v_1}^3} - \frac{3{p_2}^2\,{T_2}^2}{16\,{v_2}^3\pi^2} + \frac{11\,{T_1}^4{e_1}^4}{2\,{v_1}^4} + \frac{11{p_2}^4\,{T_2}^4}{512\,{v_2}^4\pi^4}\,,$$

$$L_2 = -\frac{1}{16}\boldsymbol{R}_{klmn}\boldsymbol{H}_p{}^{kl}\boldsymbol{H}^{pmn} = \tag{4.13}$$

$$= -\frac{{e_3}^2}{{v_1}^3\,{T_1}^2} - \frac{{p_4}^2}{16\,\pi^2\,{v_2}^3\,{T_2}^2} + \frac{{e_3}^2\,{e_1}^2}{{v_1}^4} + \frac{{p_4}^2\,{p_2}^2}{256\,\pi^4\,{v_2}^4}\,,$$

$$L_3 = -\frac{1}{64}\boldsymbol{H}_k{}^{mn}\boldsymbol{H}_{lmn}\boldsymbol{H}^{kpq}\boldsymbol{H}^l{}_{pq} = -\frac{3\,{e_3}^4}{{v_1}^4\,{T_1}^4} - \frac{3\,{p_4}^4}{256\,\pi^4\,{v_2}^4\,{T_2}^4}\,, \tag{4.14}$$

$$L_4 = \frac{1}{192}\boldsymbol{H}_{klm}\boldsymbol{H}^k{}_{pq}\boldsymbol{H}_r{}^{lp}\boldsymbol{H}^{rmq} = \frac{{e_3}^4}{2\,{v_1}^4\,{T_1}^4} + \frac{{p_4}^4}{512\,\pi^4\,{v_2}^4\,{T_2}^4}\,. \tag{4.15}$$

Inserting the above expressions in the ten dimensional action at the supergravity approximation (3.1) and its linear α' corrections (3.3) gives

$$S = \boldsymbol{S} = \frac{1}{32\pi}\int dt\,dr\,d\phi\,d\cos\theta\;s\,v_1\,v_2\,(L_0 + \alpha'(L_1 + L_2 + L_3 + L_4)) + O(\alpha'^2)\,, \tag{4.16}$$

where the integration over the compactified space has been done.[3] Then the entropy function reads

$$f(\vec{v}, \vec{T}, \vec{e}, \vec{p}, s) = \frac{1}{8}s\,v_1\,v_2\,(L_0 + \alpha'(L_1 + L_2 + L_3 + L_4)) + O(\alpha'^2)\,, \tag{4.17}$$

using which in (4.5) gives the equations of motion near the horizon. These equations should be solved perturbatively. Thus we write α' expansion series for the constant parameters labelling the horizon configuration,

$$\begin{aligned} v_i &= {v_i}^0\,(1 + \alpha'\,\tilde{v}_i + O(\alpha'^2))\,, \\ T_i &= {T_i}^0\,(1 + \alpha'\,\tilde{T}_i + O(\alpha'^2))\,, \\ e_i &= {e_i}^0\,(1 + \alpha'\,\tilde{e}_i + O(\alpha'^2))\,, \\ s &= s^0\,(1 + \alpha'\,\tilde{s} + O(\alpha'^2))\,. \end{aligned} \tag{4.18}$$

Not that the electric and magnetic charges are fixed to their values at the supergravity approximation because there exists no α' corrections to the charges of the dyonic black hole. The equations of motion (4.5) at the supergravity approximation ($\alpha' = 0$) are solved by

$$\begin{aligned} {v_1}^0 &= {v_2}^0 = \frac{p_2\,p_4}{4\,\pi^2}\,, \\ {T_1}^0 &= \sqrt{\frac{p_4}{p_2}}\,,\; {T_2}^0 = \sqrt{\frac{q_1}{q_3}}\,, \end{aligned} \tag{4.19}$$

[3]The idea of identifying the induced action near the horizon rather than in the whole of the space was used in [34] to study the linear α' corrections for singular backgrounds representing a wrapped fundamental string.

$$
\begin{aligned}
e_1{}^0 &= \frac{1}{4\pi}\sqrt{\frac{q_3\, p_2\, p_4}{q_1}}\,, \quad e_3{}^0 \;=\; \frac{1}{4\pi}\sqrt{\frac{q_1\, p_2\, p_4}{q_3}}\,,\\
s^0 &= 8\,\pi\sqrt{\frac{q_1\, q_3}{p_2\, p_4}}\,.
\end{aligned}
$$

These are the horizon configuration parameters at the supergravity approximation. Inserting (4.18) and (4.19) in (4.5) gives a set of linear algebraic equations for the linear α' corrections (4.18) to the supergravity approximation (4.19). These linear equations are solved by

$$
\tilde{v}_1 = 0\,, \tag{4.20}
$$

$$
\frac{1}{2}\tilde{v}_2 = \tilde{T}_2 = -\,\tilde{T}_1 = -\,\tilde{s} = \tilde{e}_1 = \tilde{e}_3 = \frac{\pi^2}{p_2\, p_4}\,. \tag{4.21}
$$

The modified Hawking-Bekenstein (Wald) entropy is expressed by the Legendre transformation of the entropy function

$$
S_{BH} = 2\,\pi\,(e_1\,\frac{\partial f(\vec{v},\vec{T},\vec{e},\vec{p},s)}{\partial e_1} + e_3\,\frac{\partial f(\vec{v},\vec{T},\vec{e},\vec{p},s)}{\partial e_3} - f(\vec{v},\vec{T},\vec{e},\vec{p},s))\,, \tag{4.22}
$$

evaluated on the horizon configuration [23, 24]. Inserting (4.18) and (4.19) in (4.22) we get

$$
S_{BH} = \sqrt{p_2\, p_4\, q_1\, q_3}\,(1 + \frac{\pi^2\,\alpha'}{p_2\, p_4}) + O(\alpha'^2)\,. \tag{4.23}
$$

We notice that Wald entropy does not depend on the values of $\tilde{v}_1$, $\tilde{v}_2$, $\tilde{T}_1$, $\tilde{T}_2$, $\tilde{e}_1$, $\tilde{e}_2$ and $\tilde{s}$. This is due to the fact that the α' corrections can be compensated by a perturbative field redefinition [25] and Wald entropy is independent of the field redefinition ambiguities [35]. The Independence of the entropy on the values of $\tilde{v}_1, \cdots, \tilde{s}$ is a direct consistency check of our solution.

Note that (4.23) is computed by doing perturbation in $\frac{\alpha'}{p_2\, p_4}$. Therefore we could not extrapolate (4.23) to the case of a wrapped fundamental string as an extremal dyons with zero-magnetic charges. Fundamental strings are null singular at the supergravity approximation [36]. It is suggested that the α' corrections convert these singular solutions to black holes [37]. However these black holes are realised as the exact solutions of the truncated α'-corrected equations of motion. The exact solutions of the truncated equations and their associated entropy are scheme dependent. It was showed [30] that there exist schemes in which the inclusion of all the linear α' corrections produces a horizon for which Wald entropy is in agreement with the statistical entropy. Ref. [30] has improved the conclusions of [37] and many follow works where a part of the linear α' corrections in some specific schemes were studied. However it still remains disturbing that the entropy is scheme dependent. Comparison of the entropy of a regular dyon with the entropy of a fundamental string requires first finding the exact solutions corresponded to the truncated entropy function in a general scheme considered in [30] and next going to the limit of vanishing magnetic charges. Here we suffice to conclude that (4.23) being valid for large p_2 and p_4 is not necessarily in contradiction with the limit of vanishing p_2 and p_4 studied in [30, 37].

In the following we would like to compare all the linear α' corrections to the entropy (4.23) with the corrections given by only the inclusion of the Gauss-Bonnet Lagrangian in the induced action [17]. The square of the Riemann tensor of the four dimensional metric is

$$\frac{1}{8}\, R_{ijkl}\, R^{ijkl} = \frac{1}{2\, {v_1}^2} + \frac{1}{2\, {v_2}^2}\,. \tag{4.24}$$

We see that (4.24) coincides with the first two terms in L_1. In total ten terms in $L_1 + L_2 + L_3 + L_4$ are not given by the square of the Riemann tensor. The Gauss-Bonnet Lagrangian in the four dimensions is

$$L_{GB} = R_{ijkl}\, R^{ijkl} \;-\; 4\, R_{ij}\, R^{ij} \;+\; R^2 \;=\; -\,\frac{8}{v_1\, v_2}\,. \tag{4.25}$$

Including the Gauss-Bonnet action to the induced action at the supergravity approximation is equal to including (4.24) and performing a field redefinition. Thus ten terms in the linear α' corrections to the induced action are not produced by the inclusion of the Gauss-Bonnet action. The inclusion of the Gauss-Bonnet action in the induced action at supergravity approximation gives the following entropy function

$$f^\star = \frac{1}{8}\, s^\star\, {v_1}^\star\, {v_2}^\star\, (L_0 \;+\; \frac{\alpha'}{8}\, L_{GB}) \;=\; \frac{1}{8}\, s^\star\, {v_1}^\star\, {v_2}^\star\, L_0 \;-\; \frac{\alpha'}{8}\, s^\star \;+\; O(\alpha')\,, \tag{4.26}$$

where we used * to distinguish the near horizon parameters identified by (4.26) with those identified by (4.17). This entropy function (4.26) identifies the horizon configuration parameters to

$$\frac{{v_i}^\star}{{v_i}^0} \;=\; \frac{{e_i}^\star}{{e_i}^0} \;=\; 1 \;+\; \frac{2\,\pi^2\,\alpha'}{p_2\, p_4}\,,\;\; \frac{{T_i}^\star}{{T_i}^0} = 1\,,\;\; \frac{s^\star}{s^0} \;=\; 1 \;-\; \frac{2\pi^2\,\alpha'}{p_2\, p_4} \tag{4.27}$$

where v_1^0, $\cdots$, s^0 are given by (4.19) and for which the entropy reads

$$S_{GB} = \sqrt{p_2\, p_4\, q_1\, q_3}\, (1 \;+\; \frac{2\,\pi^2\,\alpha'}{p_2\, p_4}) \;+\; O(\alpha')\,, \tag{4.28}$$

Ref. [17] has included the Gauss-Bonnet action in the induced action at the supergravity approximation and has solved the corresponding truncated α'-corrected equations of motion exactly. We note that (4.28) is in agreement with the large charge expansion of eq. (3.13) of ref. [17] after setting $n = 2\, q_1$, $w = 2\, q_3$, $\tilde{N} = \frac{p_2}{4\,\pi}$, $\tilde{W} = \frac{p_4}{4\,\pi}$ and using the unit of $\alpha' = 16$.

We notice that (4.28) is not in agreement with (4.23) therefore we conclude that all the linear α' corrections to the entropy of a regular dyon with a large horizon can not be reproduced by inclusion of only the Gauss-Bonnet Lagrangian to the Lagrangian density at the supergravity approximation.

Ref. [17] argues[4] that inclusion of only the Gauss-Bonnet Lagrangian to the supergravity approximation is agreement with inclusion of the supersymmetric version of the square of the Riemann tensor [7]-[14] which in turn is in agreement with the proposal for the statistical entropy [1]-[6]. Ref. [1]-[6] provides an index which counts the number of bosons minus fermions. The entropy counts the total number of states. It happens that the index agrees with leading order entropy of a dyonic black hole and on inclusion of only the Gauss-Bonnet or the supersymmetric version of the Riemann square terms.[5] The disagreement between (4.23) and (4.28) suggests that the map from the index defined in [1]-[6] to the thermodynamical entropy should be modified at the linear order in α' at least for the case of regular dyons with a large event horizon.

5. Conclusions

We have computed all the linear α' corrections to the thermodynamical entropy for a four dimensional dyonic black hole carrying arbitrary momentum, winding, KK-monopole and H-monopole charges in the toroidal compactification of the Heterotic string theory.

We have seen that all the linear α' corrections to the entropy of a regular dyon with a large horizon can not be reproduced by inclusion of only the Gauss-Bonnet Lagrangian to the Lagrangian density at the supergravity approximation which in turn is in agreement with the statistical the entropy [1]-[6]. Ref. [1]-[6] provides an index which counts the number of bosons minus fermions. The entropy counts the total number of states. The disagreement between all the linear α' corrections to the entropy (4.23) and the corrections given by the inclusion of only the Gauss-Bonnet terms (4.28) suggests that the map from the index defined in [1]-[6] to the thermodynamical entropy should be modified at the linear order in α' at least for the case of dyonic black holes with a large event horizon.

We have studied the α' corrections as perturbation on a given black hole geometry in an asymptotically flat space time. We have showed that in general the existence

[4]The written argument in ref. [17] is based on going to the limit of vanishing p_2 and p_4. The corrections to the entropy due to the inclusion of the Guess-Bonnet action or the supersymmetric Riemann squared terms may not be the same for large values of p_2 and p_4. I am not familiar with the notation used in the literature of the supersymmetric version of the Riemann squared term. Therefore I have not directly compared the supersymmetric Riemann squared term versus all the linear α' corrections. I suffice to presume the validity of the conclusion of [17].

[5]Justin David, private communication on the statistical entropy for dyons and on ref. [1]-[6].

of a smooth α' corrections on and outside the horizon requires to alter the fall off of the fields at asymptotic infinity. Thus the charges may receive α' corrections. The attractor equations and the entropy formalism do not answer if (and how much) the charges are corrected. Therefore generically the attractor equations [21, 22] and the entropy formalism [23, 24] do not suffice to express the parameters of the horizon configuration in terms of the values of the charges at the supergravity approximation. We have showed that the charges of the dyonic black retain their values at the supergravity approximation. Therefore one can use the entropy formalism to evaluate the linear order α' corrections in the modified Hawking-Bekenstein entropy for dyons.

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